

Need versus Merit: The Large Core of College Admissions Markets*

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Abstract

This paper studies the set of stable allocations in college admissions markets where students can attend the same college under different financial terms. The stable deferred acceptance mechanism implicitly allocates funding based on merit. In Hungary, where the centralized mechanism is based on deferred acceptance, an alternate stable algorithm would change the assignment of 9.3 percent of the applicants, and increase the number of assigned applicants by 2 percent. Low socioeconomic status applicants and colleges in the periphery benefit disproportionately from moving to this non-merit-based algorithm. These findings stand in sharp contrast to findings from the matching (without contracts) literature.

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1 Introduction

In recent years, a growing number of students are being assigned to schools through centralized clearinghouses. The success of such clearinghouses crucially relies on the use of a stable matching mechanism (Roth and Xing, 1994; Roth, 2002).¹ The matching market design literature finds that a designer who wishes to implement a stable allocation has limited scope for further design. First, the rural hospital theorem determines that the same positions are filled in all stable allocations (Roth, 1984a, 1986). Second, the set of stable allocations has the consensus property: all students prefer the outcome of the student-proposing deferred acceptance mechanism (henceforth SP-DA) to any other stable allocation (Gale and Shapley, 1962; Roth, 1984b). Third, empirical and theoretical studies suggest that all students, save for a handful, receive the same assignment in all stable allocations (e.g., Ashlagi et al., 2017; Azevedo and Leshno, 2016; Immorlica and Mahdian, 2005; Kojima and Pathak, 2009; Roth and Peranson, 1999). This last finding implies that students and schools have limited incentives to collect information, and that incentives to misreport one’s preferences to SP-DA are minimal (Demange et al., 1987).

The above-mentioned results apply to two-sided matching markets (men and women, students and schools, etc.) where agents’ preferences are over potential partners from the other side. However, the environments studied and designed by economists are often more complex. For example, college applicants care not only about the study program they are assigned to, but also about the level of financial aid they receive. In this paper, we ask whether the set of stable allocations continues to be small in these more complex environments.

We study Hungarian college admissions, where colleges offer multiple levels of financial aid and the centralized clearinghouse uses a variant of deferred acceptance (DA). We develop an alternate stable algorithm and apply it to administrative data from the centralized clearinghouse in 2007. We show that relative to the algorithm in place, our stable algorithm would change the assignment of more than 10,000 applicants (approximately 10 percent of the applicants), with slightly more winners than losers. More importantly, our alternate stable algorithm would increase the number of applicants assigned to college by more than 2 percent (approximately 1,700) with approximately 2,300 unassigned applicants gaining admission and approximately 600 losing their place. Since programs in the capital—which are generally prestigious and highly demanded—typically fill all their positions both under DA and under our alternative, the gains in enrollment mostly accrue to colleges in the periphery. Our findings demonstrate that the consensus property and the rural hospital theorem do not extend to this more complex environment and do not even hold approximately.

Our finding that different stable allocations differ substantially stands in sharp contrast to those of Roth and Peranson (1999). They find that only about 0.1% of approximately 20,000 applicants to the National Residency Match Program (NRMP) in the early 1990s would have received a different assignment had the algorithm been changed from student-receiving (i.e., hospital-proposing) to student-proposing.² Like Roth and Peranson, we also find that all but 8 applicants (approximately 0.01%) receive the same assignment

¹Stability is also useful for predicting behavior in decentralized matching markets (e.g., Banerjee et al., 2013).

²Similar findings are presented by Banerjee et al. (2013) in Indian marriage markets, and by Hitsch et al. (2010) in an online dating market.

under student-proposing and student-receiving DA.

We analyze the characteristics of those who benefit and lose from our alternate stable algorithm. Applicants who prefer the outcome of our algorithm to the outcome of DA come from a lower socioeconomic background relative to applicants who prefer the outcome of DA. Our alternative stable allocation also increases geographic mobility by increasing the number of applicants assigned to a college outside their county of residence.

Our alternate stable algorithm can be interpreted as allowing colleges to exercise their local market power. To gain intuition, we provide a simple example. The example is a minimal instance of our model of Hungarian college admissions.³ We accompany the verbal description with the notation of our model, which we introduce in Section 3.

Example 1. There are two students, $S = \{r, p\}$, and one college, $C = \{c\}$. The college has two seats, but only one of these seats is state-funded ($q_c^0 = 1, q_c^1 = 1$). The college finds the rich student, r , more attractive than the poor student, p (i.e., $r \gg_c p$). The college prefers to accept the most attractive students, and to fill its capacity. The rest of the college's preferences over acceptable allocations are fully described by

$$\{(r, c, 0), (p, c, 1)\} \succ_c \{(r, c, 1), (p, c, 0)\} \succ_c \{(r, c, 0)\} \succ_c \{(r, c, 1)\} \succ_c \{(p, c, 0)\} \succ_c \{(p, c, 1)\},$$

where 1 (0) indicates admission with (without) state funding.

The rich student, r , prefers to receive state funding, but she is willing to attend c even if she does not get state funding. Formally, r 's preferences are given by $(r, c, 1) \succ_r (r, c, 0) \succ_r \emptyset$. By contrast, the poor student, p , is only interested in admission with state funding. Thus, p 's preferences are summarized by $(p, c, 1) \succ_p \emptyset$.

There are two stable allocations in this market: $\{(r, c, 1)\}$, which is the result of SP-DA, and $\{(r, c, 0), (p, c, 1)\}$. The only allocation that both students weakly prefer to both of these stable allocations has both of them receiving state funding, but this allocation is not acceptable to the college.

Given a stable allocation, colleges have *local market power* over students who are admitted with state funding, but who have no outside option (a contract at another college or being unassigned) that they prefer to the self-funded contract with the same college. An extreme case is when students rank the state-funded and self-funded contracts with the same college consecutively, like the rich student in our example. In this case, colleges can exercise market power by refusing to accept such students to state-funded seats, thus freeing up the state-funded seats which can then be used to recruit price-sensitive students, like the poor student in our example.⁴

In the example, SP-DA assigns the state-funded seat, which both students prefer, to the more attractive student. This is not a coincidence: we show that under SP-DA, as well

³Our model captures the structure of preferences observed in other centralized college admissions markets, including Turkey, Australia, Israel, Ukraine, Russia, and the US (Akar, 2010; Artemov et al., 2017; Hassidim et al., 2017b; Kiselgof, 2011; Peranson, 2019).

⁴This argument is not precise, since by recruiting new students the college may change the outside options available to its other students. Our empirical analysis takes this challenge into account.

as under student-receiving DA (henceforth, SR-DA), funding is allocated based on merit (Proposition 5). By contrast, funding is not allocated based on merit in the other stable allocation of our example.

Our findings speak to the broader research question of how to design college admissions processes in the presence of constraints on additional resources such as financial aid, dormitories, etc. Our analysis does not consider many policy levers that are available to policymakers. For example, we fix the number of scholarships the government allocates to each program, we keep scholarships indivisible, and we require that admission and financial terms be determined simultaneously based on preference reports and that the outcome is stable. Even under these restrictions, we find a substantial scope for market design to affect the outcome.

Our findings suggest that when colleges are free to act strategically, they have an incentive to offer financial aid selectively, based on students' outside options. Since students' outside options depend on their preferences and on the behavior of other agents (students and colleges), information on other agents is crucial for colleges in order to successfully implement this strategy (cf. [Azevedo and Budish, 2018](#)). [Caniglia and Porterfield \(forthcoming\)](#) show that using information on students' family finances is useful to this end.⁵ Alternatively, colleges can use an early decision policy as a screening device in order to exert their market power ([Avery et al., 2009](#); [Ehrenberg, 2009](#); [Kim, 2010](#); [Wang and Zhou, 2018](#)).

Our results shed light on the policy debate on market power in higher education (see, e.g., [Hoxby, 2000](#)). This literature gained traction after the U.S. Department of Justice (DoJ) brought an antitrust case against a group of elite colleges for sharing prospective students' financial information and coordinating their financial aid policy. MIT contested the charges, claiming that this practice prevents bidding wars over the best students and thus frees up funds to support needy students, and that MIT does not profit financially from this practice ([DePalma, 1992](#)). In 1994, Congress passed the Improving America's Schools Act, whose Section 568 permits some coordination and the sharing of information between institutions with a need-blind admissions policy. Our findings provide support to MIT's arguments. We show that even in the absence of a motive to increase profit, colleges have an incentive to apply market power in order to improve the quality of their incoming cohorts, and that the consequences for students are heterogeneous. On average, needy students gain, in part at the expense of more wealthy students. Furthermore, information about students' outside options is necessary to facilitate such behavior.

The remainder of the paper is organized as follows. Following a short review of related literature, Section 2 describes college admissions in Hungary. Section 3 presents a formal model of Hungarian college admissions, state our theoretical results, and presents our alternate algorithms. Proofs are relegated to the appendix. Section 4 describes our data and presents summary statistics. Section 5 presents the empirical findings. Section 6 concludes.

⁵[Caniglia and Porterfield \(forthcoming\)](#) analyze Franklin & Marshall College's move from merit- to need-based financial aid. They find that reallocating financial aid to needy students had no effect on the probability of non-needy students accepting an offer of admission, but that it had a sizable effect on the probability of needy students accepting such an offer. This allowed the college to improve the "quality" of the incoming cohorts, and to diversify its student body.

1.1 Related Literature

Empirical and theoretical studies show that the set of stable allocations is typically small (Ashlagi et al., 2017; Azevedo and Leshno, 2016; Immorlica and Mahdian, 2005; Kojima and Pathak, 2009; Roth and Peranson, 1999; Storms, 2013). Prior to these studies, a large portion of the literature on the theory of two-sided matching markets was motivated by the potential multiplicity of stable allocations. Examples include studies of the structure of the set of stable allocations (Knuth, 1976), of fair stable allocations (Klaus and Klijn, 2006; Schwarz and Yenmez, 2011), and of incentives (Ehlers and Massó, 2007; Roth, 1982; Sönmez, 1999). The rural hospital theorem (Roth, 1984a, 1986) refuted suggestions that changing the way the National Residency Match Program (NRMP) treats medical graduates and hospitals may change the number of doctors assigned to rural hospitals.

Truthful reporting to the student-proposing DA mechanism is a weakly dominant strategy for students, and there is no stable matching mechanism that makes truthful reporting dominant for both sides of the market (Dubins and Freedman, 1981; Roth, 1982). Demange et al. (1987) show that schools' incentives to manipulate SP-DA are intimately related to the multiplicity of stable allocations.⁶ Several studies show that in large markets it is safe for schools to report their true preferences to SP-DA (Ashlagi et al., 2017; Azevedo and Budish, 2018; Immorlica and Mahdian, 2005; Kojima and Pathak, 2009; Lee, 2016).

Complex two-sided matching markets are studied in the matching-with-contracts literature (Hatfield and Milgrom, 2005). Much of this literature focuses on identifying conditions under which SP-DA remains stable and strategy-proof for students. Examples include Fleiner (2003), Hatfield and Kojima (2010), Hatfield and Kominers (2015), Hatfield and Milgrom (2005), Hatfield et al. (forthcoming), Kelso and Crawford (1982), and Roth (1984b). Applications related to college admissions include Abizada (2016), Afacan (2020), Aygün and Bó (2016), Nei and Pakzad-Hurson (2016), Pakzad-Hurson (2014), Westkamp (2013), and Yenmez (2018). We contribute to this literature by studying the size of the set of stable allocations in a matching-with-contracts market.

Hassidim et al. (2017a) show that the theoretical results on the size of the set of stable allocations do not extend to certain settings with contracts. Rheingans-Yoo (2020) expands some of their findings to settings where the preference structure is localized.

Finally, our paper is related to studies of reserve design (Dur et al., forthcoming, 2018; Pathak et al., 2020a,b). In the context of school choice, a key observation in this literature is that students are indifferent between different seats in the same school, but some seats are reserved for certain groups of students. This implies, using our terminology, that schools have local market power over all assigned students. The reserve-design literature focuses on the effect of different ways the mechanism can break students' preference ties in order to form strict student rankings of contracts, while keeping the priorities at each seat fixed. By contrast, we study an environment where students have strict preferences over all contracts.

⁶Numerous studies have analyzed the optimal behavior of schools when the SP-DA mechanism is in place (e.g., Coles and Shorrer, 2014; Ehlers, 2004; Konishi and Ünver, 2006; Roth and Rothblum, 1999; Sönmez, 1997).

2 Background: College Admissions in Hungary

Each year about 80,000 applicants are assigned to undergraduate college programs in Hungary. Applicants are assigned to *programs*, i.e., a specific major in a specific college.⁷ The admissions process has been centralized since 1985. The centralized clearinghouse has been using variants of DA to assign applicants to colleges.

Historically, higher education in Hungary was free. However, in recent years, the government has capped the number of state-funded (free) seats, and programs are allowed to offer admission to self-funded (tuition-paying) seats as well.⁸ To implement this change, the centralized clearinghouse started requesting applicants to submit a rank-order list (ROL) of *alternatives*, specifying that the student attends a program under particular financial terms (e.g., economics in the University of Debrecen with state funding). This flexibility was crucial in order to allow applicants to express their preferences. For example, some applicants may not be willing or able to afford paying tuition, while others may have a strong preference for certain programs, and be less price-sensitive.

Applicants may rank as many alternatives as they wish. Submitting an ROL with three programs (corresponding to up to six alternatives) only requires paying the fixed application fee of \$50 (9,000HUF). Applicants are required to pay a registration fee of \$11 (2,000HUF) for each additional program in their ROL.

College programs report to the mechanism their capacity (i.e., the maximal number of applicants they can accept). Admissions priorities in each program are based on a weighted average of several variables (mainly academic performance in the 11th and 12th grades and matriculation exam scores, but also credits for disadvantaged and disabled applicants, for applicants who demonstrate fluency in another language, and for a small number of gifted applicants). Different study programs may use different weighting schemes (e.g., a computer science program may assign a greater weight to physics grades relative to a psychology program).

In 2007, the year we focus on, to accommodate the multiple modes of financing, the mechanism required that the programs also report the number of state-funded seats they offered. This took place prior to the beginning of the admissions process. The mechanism then created two “auxiliary programs” with capacities corresponding to the number of state-funded and self-funded seats in the program, and endowed both with the priorities of the program.⁹ Finally, student-receiving DA (SR-DA)—i.e., auxiliary-program-proposing DA—was used to determine the final assignment.

⁷We focus on admissions to undergraduate programs, which include three types of programs: bachelor’s degree programs (with a typical study duration of 3 years), combined bachelor–master’s programs (with a typical study duration of 5 years), and tertiary vocational education (with a typical study duration of 2 years).

⁸Citizens of the European Economic Area who have not yet graduated from higher education are eligible for state funding.

⁹This description applies to full-time programs. There was an additional constraint on the total number of state-funded seats in each field of study, which was only binding for part-time programs. Additionally, part-time programs in computer science and engineering use slightly different priorities for state-funded and self-funded seats (they assign a lower weight to mathematics scores of applicants for self-funded seats relative to state-funded seats). Our empirical analysis takes this into account (see Appendix E).

Timeline. The application process proceeds as follows. First, the centralized clearinghouse publishes a booklet that includes the rules of the college admissions process together with the list of the alternatives (program–financial terms pairs), and the number of seats available in each alternative in November. Then, applicants submit their ROLs in mid-February. Finally, in mid-July, the clearinghouse notifies applicants about their placement and publishes the priority-score cutoffs for each alternative, i.e., the minimum priority score that was needed to gain admission.

Financing Colleges. The government funds colleges on a per-student basis, irrespective of the financial terms. Additionally, colleges collect tuition from students with self funding,¹⁰ and receive additional compensation for state-funded students from the government. Thus, holding the number of students with state funding (self funding) constant, colleges’ revenue increases with each additional admitted self-funded (state-funded) student.

Table 1 presents information on the alternatives that colleges offered in 2007. The applicants’ choice set consisted of 3,740 alternatives, corresponding to 2,289 study programs. The number of study programs that offered both state-funded and self-funded alternatives was 1,451. Some study programs were available with state-funding exclusively (172), and other programs—mostly part-time programs—were available with self-funding exclusively (666). Panel B shows that 93,999 of the 111,685 available seats (84.1 percent) were available in study programs that offered both state-funded and self-funded alternatives. The overwhelming majority of the seats that were offered with self-funding exclusively (about 91 percent) were in the form of part-time education.

Table 1: Applicants’ choice set

| | Total (1) | Full-time (2) | Part-time (3) |
|------------------------------|--------------|------------------|------------------|
| <i>A. Alternatives</i> | | | |
| Number of programs | 2,289 | 1,357 | 932 |
| – state- and self-funded | 1,451 | 1,133 | 318 |
| – state-funded exclusively | 172 | 158 | 14 |
| – self-funded exclusively | 666 | 66 | 600 |
| Number of alternatives | 3,740 | 2,490 | 1,250 |
| <i>B. Capacities</i> | | | |
| State-funded and self-funded | 93,999 | 75,197 | 18,802 |
| – state-funded | 47,809 | 43,084 | 4,725 |
| – self-funded | 46,190 | 32,113 | 14,077 |
| State-funded exclusively | 917 | 806 | 111 |
| Self-funded exclusively | 16,769 | 691 | 16,078 |
| Total | 111,685 | 76,694 | 34,991 |

Notes: The table presents summary statistics of the applicants’ choice set. Panel A presents the number of alternatives by the financial terms. Panel B displays the number of seats by the financial terms.

¹⁰Tuition for a 3-year bachelor’s degree program ranges from \$3,280 to \$6,560 (600,000–1,200,000HUF).

3 A Model of Hungarian College Admissions

We use the many-to-one matching-with-contracts model of [Hatfield and Milgrom \(2005\)](#) to describe *Hungarian college admissions markets*. There is a finite set of *colleges*, C , a finite set of *students*, S , and a set of *financial terms*, $T = \{0, 1\}$. A *contract* is a tuple $(s, c, t) \in S \times C \times T$ that specifies a student, a college, and financial terms (with $t = 1$ representing state funding, and $t = 0$ representing self funding).

An *allocation* is a subset $Y \subseteq S \times C \times T$. An allocation Y is *feasible* if, no student is included in more than one contract. Formally, for each student s , $|Y \cap (\{s\} \times C \times T)| \leq 1$. Given an allocation, Y , we let Y_S denote the set of students involved in some contract in Y . Formally, $Y_S := \{s \in S \mid Y \cap (\{s\} \times C \times T) \neq \emptyset\}$. Similarly, $Y_C := \{c \in C \mid Y \cap (S \times \{c\} \times T) \neq \emptyset\}$.

Students' preferences Each student, s , has strict preferences over contracts in $\{s\} \times C \times T$ (not all of them are necessarily available) and an outside option which we denote by \emptyset . We denote student s 's preference relation by \succ_s . Students' preferences, therefore, induce weak preferences over all feasible allocations, where students only consider their own assignment.

Colleges' preferences Each college, c , has strict preferences, \succ_c , over all feasible allocations in $S \times \{c\} \times T$.¹¹ These preferences induce weak preferences over all feasible allocations, where c only considers contracts in $S \times \{c\} \times T$. The preferences of each college, c , satisfy the following conditions.

First, c is associated with two numbers, q_c^1 and q_c^0 , representing a constraint on the number of students that can be accepted under each of the financial terms. The college c prefers the empty allocation to all allocations that *violate c 's quotas*, that is, that assign to c more than q_c^t students under the financial terms t .

Second, c has a complete order over $S \cup \{\emptyset\}$, denoted by \gg_c , representing a ranking over students. Given an allocation $Y \subseteq S \times \{c\} \times T$ (an assignment of students to the college under some financial terms), if q_c^t is not binding then the college prefers to accept an additional student s under the financial terms t if and only if $s \gg_c \emptyset$. Formally, if $s \notin Y_S$ and $|Y \cap (S \times \{c\} \times \{t\})| < q_c^t$, then $Y \cup \{(s, c, t)\} \succ_c Y$ iff $s \gg_c \emptyset$. Additionally, c prefers to replace student s who receives the financial terms t with another student s' who is not assigned to c (under the same financial terms) if and only if $s' \gg_c s$. Formally, for all $Y \subseteq S \times \{c\} \times T$, if $(s, c, t) \in Y$ and $s' \notin Y_S$, then $(Y \cup \{(s', c, t)\}) \setminus \{(s, c, t)\} \succ_c Y$ iff $s' \gg_c s$.

Third, so long as quotas are not violated, the composition of the incoming cohort is lexicographically more important to the college relative to the way funding is allocated. Formally, let $Y, Y' \subseteq S \times \{c\} \times T$ be two feasible allocations that do not violate c 's quotas. Then if $Y_S = Y'_S$ (i.e., Y and Y' differ only in the identity of the recipients of state funding in c) and $Y \succ_c Y'' \succ_c Y'$ for some $Y'' \subseteq S \times \{c\} \times T$, it follows that $Y''_S = Y_S$.

Discussion of theoretical assumptions According to our assumptions, subject to quotas, colleges' preferences depend more on the size and quality of the incoming cohort than they do on the distribution of funding. This assumption is not unusual in the literature (e.g., [Kim, 2010](#); [Heo, 2017](#)), and is consistent with preferences of colleges in other markets (e.g., [Hassidim et al., 2017b](#)). Our assumptions rule out redistributive motives. As highlighted

¹¹None of our results relies on the assumption that colleges' preferences are strict.

by [Caniglia and Porterfield \(forthcoming\)](#), the presence of such motives will strengthen our arguments.

Our model also abstracts from upper quotas on state-funded seats in part-time programs in the same field ([Biró et al., 2010](#)). We note that all of our theoretical results generalize to this more complex environment. The upper quotas create links between part-time programs, making their choice functions more complex. The added complexity creates further opportunities to apply local market power (e.g., transfer students between part-time programs). We choose not to take advantage of this feature, which is specific to the Hungarian market, in our empirical analysis.

The special case of our model where $q_c^1 = 0$ for all c corresponds to *two-sided many-to-one matching (without contracts) markets with responsive preferences* ([Roth, 1985](#)). Given such a market, we often refer to college c 's only positive quota as c 's *quota* and to allocations as *matchings*.

Individual rationality The preferences of each college c induce a choice function $\text{Ch}_c : 2^{S \times C \times T} \rightarrow 2^{S \times \{c\} \times T}$, that identifies the feasible subset of $Y \cap (S \times \{c\} \times T)$ most preferred by c . Similarly, for students, $\text{Ch}_s : 2^{S \times C \times T} \rightarrow 2^{\{s\} \times C \times T}$ chooses the most preferred acceptable contract involving s (if one exists). An allocation Y is *individually rational* if all agents choose all the contracts in which they are involved, that is $\bigcup_{c \in C} \text{Ch}_c(Y) = \bigcup_{s \in S} \text{Ch}_s(Y) = Y$. Individually rational allocations cannot violate quotas since colleges prefer the empty set to allocations that violate their quotas. Furthermore, they are feasible since students never choose more than one contract ($|\text{Ch}_s(\cdot)| \leq 1$ for all s).

Stability An allocation Y is *blocked (through Z)* if there exists a college, c , and a non-empty set $Z \subseteq (S \times \{c\} \times T) \setminus Y$ such that $Z \subseteq \text{Ch}_c(Y \cup Z)$ and $Z \subseteq \bigcup_{s \in S} \text{Ch}_s(Y \cup Z)$. An allocation is *stable* if it is individually rational and not blocked. In words, an allocation is stable if no coalition of agents can achieve a weak improvement on its own.

Deferred acceptance Student-proposing deferred acceptance takes a profile of preferences as input and outputs an allocation. The allocation is the result of the following process. In each round, each student proposes the most preferred acceptable contract from which she has not yet been rejected, if such a contract exists. Each college then rejects all but the most preferred subset of contracts from these proposals. The algorithm terminates when no proposal is rejected, and the output is the set of contracts proposed in the last round. Given a Hungarian college admissions market, we denote the outcome of SP-DA by $Y^{\text{SP-DA}}$.

Student-receiving deferred acceptance outputs an allocation based on a different process. In each round, each college, c , proposes contracts of the form (s, c, t) that were not previously rejected to the highest-ranked (up to q_c^t) acceptable students according to \gg_c for each $t \in \{0, 1\}$. Each student then rejects all but the most preferred acceptable contract that was proposed to her, if such a contract exists. The algorithm terminates when no proposal is rejected, and the output is the set of contracts proposed in the last round. Of note, if a college proposes multiple contracts to the same student, the student must reject one of these contracts, guaranteeing the feasibility of the resulting allocation. Given a Hungarian college admissions market, we denote the outcome of SR-DA by $Y^{\text{SR-DA}}$.

In two-sided many-to-one matching (without contracts) markets with responsive preferences, SR-DA is known as college-proposing DA. We intentionally avoid this label to highlight the fact that colleges are not necessarily proposing the most preferred subset of contracts that were not rejected. Since colleges' preferences are not substitutable (Hatfield and Milgrom, 2005), such a process would sometimes “renege” on proposals that were not rejected due to the rejection of complementary contracts.

Definition 1. Let $\langle S, C, \{\succ_c, \gg_c, q_c^0, q_c^1\}_{c \in C}, \{\succ_s\}_{s \in S} \rangle$ be a Hungarian college admissions market. The *related (two-sided) matching market (without contracts)* comprises the set of auxiliary colleges $\hat{C} \equiv C \times T$ with responsive preferences, where $(c, t) \in \hat{C}$ has quota q_c^t and uses the ranking \gg_c , and the set of students S with preferences $\{\succ_s\}_{s \in S}$, such that for every $s \in S$, $c \in C$, and $t \in T$, $(c, t) \succ_s (c', t')$ if and only if $(s, c, t) \succ_s (s, c', t')$.

Definition 2. Given a feasible allocation Y , the *corresponding matching (in the related matching market)* has s matched with (c, t) iff $(s, c, t) \in Y$.

Lemma 1. *The corresponding matching of Y^{SP-DA} (Y^{SR-DA}) in the related market is the student-optimal (college-optimal) stable matching.*

Lemma 2. *If a feasible allocation is blocked in a Hungarian college admissions market, then the corresponding matching is blocked in the related matching market.*

3.1 Theoretical Results

In this section, we present our theoretical results. We begin by noting that some properties of the college admissions market studied by Gale and Shapley (1962) continue to hold in the more complex, Hungarian college admissions environment. Notably, we show that both SP-DA and SR-DA terminate in a stable allocation.

Proposition 1 (Existence). *The set of stable allocations is nonempty in Hungarian college admissions markets. In particular, Y^{SP-DA} and Y^{SR-DA} are stable.*

Proposition 2 (Weak rural hospital theorem). *In Hungarian college admissions markets:*

1. *The set of students assigned to some college is identical under SP-DA and SR-DA. Formally, $[Y^{SP-DA}]_S = [Y^{SR-DA}]_S$.*
2. *For each $c \in C$ the number of state-funded (self-funded) students assigned to c is equal under Y^{SP-DA} and Y^{SR-DA} . Formally, $|Y^{SP-DA} \cap (S \times \{c\} \times \{t\})| = |Y^{SR-DA} \cap (S \times \{c\} \times \{t\})|$ for each $c \in C$ and $t \in \{0, 1\}$.*
3. *If the college c does not fill one of its quotas under SP-DA, the same students are assigned to c under these financial terms under SR-DA. Formally, if $|Y^{SP-DA} \cap (S \times \{c\} \times \{t\})| < q_c^t$ then $Y^{SP-DA} \cap (S \times \{c\} \times \{t\}) = Y^{SR-DA} \cap (S \times \{c\} \times \{t\})$.*

Proposition 3 (Weak consensus property). *In Hungarian college admissions markets, all students weakly prefer Y^{SP-DA} to Y^{SR-DA} .*

Proposition 4 (Strategic properties). *In Hungarian college admissions markets:*

1. *SP-DA is strategy-proof for students.*
2. *A student can manipulate SR-DA if and only if she strictly prefers Y^{SP-DA} to Y^{SR-DA} .*

Next, we formalize the statement that under SP-DA and SR-DA state funding is distributed according to merit. Proposition 5 implies that if two students are assigned to the same college, c , under SP-DA (SR-DA), and both students prefer a state-funded seat to a self-funded seat, then the student ranked lower according to \gg_c receives state funding only if the higher-ranked student also receives state funding.

Proposition 5. *Let s and s' be two students in a Hungarian college admissions market such that (s, c, t) and (s', c, t') belong to Y^{SP-DA} (Y^{SR-DA}) for some $c \in C$ and $t, t' \in T$. Then $s \gg_c s'$ implies that s weakly prefers (s, c, t) to (s, c, t') .*

According to Proposition 5, the outcomes of SP-DA and SR-DA have a property similar to justified-envy-freeness (Abdulkadiroğlu and Sönmez, 2003): higher-ranked students never envy the financial terms of lower-ranked students assigned to the same college as them. This property does not hold for all stable allocations (as illustrated by Example 1).¹²

We next highlight key differences between our model and the matching-without-contracts environment studied by Gale and Shapley (1962).

Proposition 6. *In Hungarian college admissions markets students may disagree on the most preferred stable allocation. Furthermore, different stable allocations may have more or fewer assigned students than Y^{SP-DA} .*

The following lemma provides a complete characterization of stable allocations.

Lemma 3. *An Allocation Y is stable if and only if all of the following hold:*

1. *Y is individually rational.*
2. *Y is not blocked through a singleton $\{(s, c, t)\}$.*
3. *Y is not blocked through a change in the financial terms of one student, and a contract with a new student using the freed-up space. Formally, for all $(s, c, 1 - t) \notin Y$ and $(s', c, t) \in Y$, Y is not blocked through $\{(s', c, 1 - t), (s, c, t)\}$.*
4. *Y is not blocked through a change in financial terms that keeps students in the same college (but under different financial terms). Formally, Y is not blocked through Z such that $(s, c, t) \in Z$ implies $(s', c, 1 - t) \in Y$.*

We note that Conditions 3 and 4 of Lemma 3 can only be violated in the presence of multiple contractual terms. Therefore, in two-sided many-to-one matching (without-contracts) markets with responsive preferences the lemma reduces to the statement that a matching is stable if and only if it is individually rational and not blocked by a student–college pair.

¹²Romm et al. (2020) study the relation between stability and the elimination of justified envy in a general matching with contracts markets.

Definition 3. Allocation Y is *certainly stable* if Conditions 1–3 of Lemma 3 hold, as well as the following condition:

- 4'. For each $(s, c, t) \in Y$, if $(s, c, 1 - t) \succ_s (s, c, t)$ then $|Y \cap (S \times \{c\} \times \{1 - t\})| = q_c^{1-t}$ and $(s', c, 1 - t) \succ_{s'} (s', c, t)$ for all $(s', c, 1 - t) \in Y$.

Remark 1. When Conditions 1-3 of Lemma 3 hold, Condition 4' of Definition 3 implies Condition 4 of Lemma 3. Therefore, a certainly stable allocation is also stable. The notion of certain stability proves useful in our empirical analysis because it only relies on quotas and rankings (i.e., q_c^0 , q_c^1 , and \gg_c) but not on the full description of \succ_c , and because there are fewer conditions to verify (each condition refers to no more than three agents).

Hassidim et al. (2017a) study more general college admissions environments that accommodate the Hungarian college admissions setting. Using the fact that colleges' preferences satisfy the hidden substitutes condition of Hatfield and Kominers (2015), they show that SP-DA is stable and strategy-proof and that different stable allocations may have different cardinality. Our proofs rely on a different construction, which allows us to derive further results. Our theoretical results contribute to Hatfield and Kominers in several ways. First, we identify a real-life market design application.¹³ Second, we provide a complete characterization of the set of stable allocations in the domain of Hungarian College admissions markets.¹⁴ Third, we show that different stable allocations may have different cardinality in this more restrictive domain. Fourth, we show that stable allocations in this domain may result in *fewer* assigned students relative to SP-DA.

3.2 Alternate Algorithms

Our theoretical findings suggest that “classic” questions in the theory of two-sided matching markets may have interesting answers in the context of Hungarian college admissions. Specifically, there may be scope for increasing the number of students in college without violating stability. We are particularly interested in assessing this scope empirically. Unfortunately, as Proposition 7 demonstrates, the problem of finding the largest stable allocation is NP-hard. This means that finding an efficient (polynomial-time) algorithm for solving the problem will establish that $P = NP$ (which is widely believed to be false).

Proposition 7. *Finding a maximum-size stable (or certainly stable) allocation in Hungarian college admissions markets is NP-hard.*

In Appendix C we show that even finding a stable allocation that gives a constant approximation to the maximum size is NP-hard. In light of this, we do not seek an algorithm that is guaranteed to be optimal. In what follows, we base our empirical analysis on two heuristics designed to increase the number of admitted students compared to SR-DA. The

¹³Colleges' preferences are not unilaterally substitutable (Hassidim et al., 2019; Hatfield and Kojima, 2010), nor do they meet any other condition that guarantees the existence of a student-optimal stable allocation.

¹⁴The substitutable completions approach of Hatfield and Kominers guarantees existence, but it is not guaranteed to identify all stable allocations. Indeed, one can verify that, in Example 1, for any substitutable completion of the college's choice function, Y^{SP-DA} is the unique allocation that is stable with respect to the completion.

heuristics rely on the application of local market power; i.e., colleges refuse to allocate state funding to applicants whose best feasible alternative is attending the college without state funding, as illustrated by Example 1. We use these heuristics to construct three alternate algorithms that result in stable allocations. Since our algorithms are not optimal, the difference (in cardinality) between their results and the allocation that results from SR-DA is a lower bound for the maximal possible difference.

Heuristic 1: Preference Flip Initialize a set A to equal the set of student–program pairs such that the student ranks the self-funded contract with the program immediately after the state-funded contract. For each pair in A , flip the order of the contracts with the program in the applicant’s original ROL, so that the state-funded contract appears immediately *after* the self-funded contract, and run SP-DA on the resulting problem. If the resulting allocation is certainly stable (with respect to original preferences), stop and output this allocation. Otherwise, remove some pairs from A and repeat the process.

Heuristic 2: Greedy Reject Initialize a set B to equal the set of student–program pairs such that the student is assigned to a state-funded seat in the program under SP-DA. For each pair in B , remove from the applicant’s initial ROL the state-funded contract with the program (the student’s assignment) and run SP-DA on the resulting problem. If the resulting allocation is certainly stable (with respect to original preferences), stop and output the resulting allocation. Otherwise, remove some pairs from B and repeat the process.

The description of both heuristics intentionally leaves a degree of freedom: the choice of elements to remove from A and B . *Algorithm 1* (*Algorithm 2*) implements Heuristic 1 (Heuristic 2) by removing pairs corresponding to the highest-ranked student in some college such that the pair is in A (B) and the state-funded contract between them is part of a potentially blocking allocation as described in Lemma 3 (or a random pair, if no such pair exists). *Algorithm 3* implements both heuristics simultaneously, removing elements from A and B according to the same criteria.¹⁵

Proposition 8. *Each of Algorithms 1–3 results in a stable allocation. Furthermore, the resulting stable allocation may have more or fewer assigned students than Υ^{SP-DA} and Υ^{SR-DA} .*

Remark 2. The proof of Proposition 8 applies to any rule regarding removal of elements from A or B .

4 Data and Summary Statistics

This section describes our data and provides summary statistics. Our main data source is administrative data on the Hungarian college admissions process that include detailed information in applicants’ ROLs, their priority scores, and capacities. Summary statistics show that a substantial fraction of applicants, typically high socioeconomic status (SES) applicants, are insensitive to prices and that low SES applicants are more likely to exclusively rank state-funded contracts in their ROLs.

¹⁵A script implementing the algorithms is available in the Supplementary Material.

4.1 Data

Our main data source is an administrative dataset that contains information on the bachelor’s degree admissions process in Hungary. In particular, we observe each applicant’s ROL, the priority score for each contract in the ROL, and all the information required to (re)calculate these priority scores.¹⁶ This information includes grades in various subjects in the final two years of high school (11th and 12th grades), performance in the matriculation exams, and the number of points the applicant received for claiming a disadvantaged background.¹⁷ For each applicant we also observe gender, postal code, and a high-school identifier. For each alternative (program–terms pair) the data includes its realized priority-score cutoff (i.e., the minimal score that was required to gain admission to this alternative). We complement our data with hand-collected information on the program-specific capacities from the 2007 information booklet.

To set the stage for our main analysis, we first calculate the outcome of SR-DA. The SR-DA assignment replicates the realized assignment for all but 3.2 percent of the applicants (3,528 applicants).¹⁸ We call the resulting allocation the benchmark. Appendix Table B1 shows that the benchmark allocation is larger by 2,567 assigned applicants, and that the difference is the result of increased utilization of self-funded seats. Intuitively, the higher utilization of self-funded seats reduces the scope for increasing the size of the stable allocation using our alternate algorithms.

The second data source is the T-STAR dataset of the Hungarian Central Statistics Office. We use it to obtain settlement-level annual information on collected income taxes. In particular, we calculate the per-capita gross annual income for all 3,164 settlements in 2007 and merge it with our main administrative data based on settlement identifiers.

The third data source is the National Assessment of Basic Competencies (NABC). Following Horn (2013), we create an NABC-based SES index, which is a standardized measure that utilizes survey information of the NABC. The NABC-based SES index resembles the economic, social, and cultural status (ESCS) indicator of the OECD PISA survey. It combines three subindices: an index of parental education, an index of home possessions (number of bedrooms, mobile phones, cars, computers, books, etc.), and an index of parents’ labor-market status. For each secondary school between 2008 and 2012 we calculate the average

¹⁶Our data report up to 7 contracts from each ROL: the first 6 contracts and the contract to which the applicant is assigned. The dataset also reports the number of contracts in each ROL. We observe the complete ROL for 91.7 percent of applicants and for 93.5 percent of all ranked contracts.

¹⁷To be eligible for disadvantaged status, an applicant must have a per-capita household income that is lower than 130 percent of the minimum pension (i.e., lower than approximately \$1,500 a year).

¹⁸There are small gaps and issues that do not allow us to fully replicate the allocation. First, we only observe up to 7 contracts in each ROL (corresponding to the top 6 choices and the realized allocation). Second, we do not know the results of auditions and other specialized admissions exams that are held by a small number of programs. Third, the mechanism has rules regarding cases where there are multiple (tied) marginal applicants which are not observable to us (Biró, 2008; Biró and Kiselgof, 2015). We address this by breaking priority ties with a single lottery. Finally, the data contain some inconsistencies, for example, applicants with a priority score of zero assigned to selective programs. In our main analysis, we address these inconsistencies by fixing the assignment of these applicants to the realized assignment, and make the corresponding seats unavailable to others. Appendix E presents the results from other approaches, and shows the robustness of our findings.

value of students' NABC-based SES index, and merge this with our main administrative dataset using high-school identifiers.¹⁹

4.2 Summary Statistics

Appendix Table B2 summarizes the means and standard deviations of the background characteristics of applicants. Altogether, 108,854 applicants participated in the college admissions process in 2007. About 5 percent of the applicants claimed points for disadvantaged status. The average per-capita annual gross income in applicants' settlement of residence is \$9,900. Approximately 20 percent of the applicants lived in Budapest, the capital, 21 percent lived in one of the 18 county capitals, 33 percent lived in towns, and the remaining 26 percent lived in villages. On average, applicants' 11th-grade GPA was 3.6, on a scale of 1 to 5. About 57 percent of the applicants are female. ROLs include, on average, 3.7 contracts with 3 programs.

Table 2 presents summary statistics on the characteristics of applicants' ROLs for all applicants, and for disadvantaged and non-disadvantaged applicants separately. Each panel focuses on a particular dimension of the contracts that applicants rank. Panel A presents statistics related to funding, that is, whether applicants ranked state-funded (self-funded) contracts exclusively. Panel B focuses on study-program characteristics, that is, whether applicants ranked exclusively contracts that are in the same field of study (e.g., science, engineering, etc.), same major (e.g., physics, civil engineering, etc.), or both contracts with the same study program consecutively. Panel C focuses on preferences for institution characteristics. It shows the share of applicants that ranked exclusively contracts that are all in a single program location (settlement), at a single university, or at a single faculty of a university.

Panel A shows that 51.4 percent of applicants ranked state-funded contracts exclusively, and 18.5 percent of applicants ranked self-funded contracts exclusively. Disadvantaged applicants are 29.4 percentage points more likely to rank state-funded contracts exclusively compared to non-disadvantaged applicants. Only 2 percent of disadvantaged applicants rank self-funded contracts exclusively.²⁰ Since market power can only be exercised over applicants who rank two contracts in the same program, these patterns suggest that the direct effect of our algorithms would likely benefit disadvantaged applicants.

Panel B shows that 54.9 percent of applicants rank exclusively contracts in a single field of study, and 29.6 percent of applicants rank exclusively contracts in a single major. Non-disadvantaged applicants are more likely to rank contracts in a single field of study and in a single major. Panel B also shows that 15.5 percent of applicants submit an ROL that ranks the same study program with state-funding and self-funding consecutively. A large share of these pairs of contracts appear at the top of applicants' ROL (11.6 percent of all applicants).

Panel C shows that 49.4 percent of applicants rank exclusively contracts that are in the same settlement, 35 percent of applicants rank exclusively contracts at a single university,

¹⁹The high-school-specific average NABC-based SES index is stable over time. Between 2008 and 2012, 95% of the variation in the high-school-specific averages of the NABC-based SES index is explained by high-school fixed effects. Furthermore, each year, about one-third of the student-level variation in the NABC-based SES index is explained by high-school fixed effects.

²⁰A likely explanation is that these applicants are ineligible for funding (see [Shorrer and Sóvágó, 2018](#)).

and 26.3 percent of applicants rank exclusively contracts at a single faculty of a university. Disadvantaged applicants are less likely to express preference for institution characteristics.

Overall, these patterns suggest that disadvantaged and non-disadvantaged applicants trade off study-program characteristics (e.g., field of study, major, institution, and location) and financial terms differently. Disadvantaged applicants put relatively more weight on financial terms, while non-disadvantaged applicants are more willing to pay for certain program characteristics.

Table 2: Summary statistics on applicants' ROLs

| | All applicants (%) (1) | Non-disadvantaged (%) (2) | Disadvantaged (%) (3) | p-value ((2)=(3)) (4) |
|--|------------------------------|---------------------------------|-----------------------------|-----------------------------|
| <i>A. Preference for funding</i> | | | | |
| State-funded contract exclusively | 51.4 | 50.0 | 79.4 | 0.00 |
| Self-funded contract exclusively | 18.5 | 19.2 | 2.0 | 0.00 |
| <i>B. Preference for study characteristics</i> | | | | |
| Single field of study | 54.9 | 55.6 | 40.1 | 0.00 |
| Single major | 29.6 | 30.3 | 15.5 | 0.00 |
| Same study program consecutively | 15.5 | 15.8 | 8.3 | 0.00 |
| Same study program consecutively on the top of the ROL | 11.6 | 11.9 | 6.2 | 0.00 |
| <i>C. Preference for institution characteristics</i> | | | | |
| Single program location | 49.4 | 50.0 | 36.6 | 0.00 |
| Single university | 35.0 | 35.4 | 27.4 | 0.00 |
| Single faculty | 26.3 | 26.9 | 15.3 | 0.00 |
| # of applicants | 108,854 | 103,840 | 5,014 | |

Notes: The table reports summary statistics on applicants' ROL. Panel A shows the share of applicants who rank state-funded (self-funded) contracts exclusively. Panel B shows the share of applicants who rank exclusively contracts in a single field of study, and in a single major. Panel B also shows the share of applicants who rank the same study program with state-funding and self-funding consecutively, and who rank the same study program with state-funding and self-funding consecutively on the top of the ROL. Panel C shows the share of applicants who rank exclusively contracts that are in the same settlement (single location), at a single university (single university), and at a single faculty of a university (single university). Column (1) presents these shares for all applicants, and columns (2) and (3) report these shares for non-disadvantaged and disadvantaged applicants, respectively. We test whether these shares differ between disadvantaged and non-disadvantaged applicants. Column (4) reports the corresponding p-values.

5 Empirical Findings

This section presents our main empirical findings. We find that, relative to SR-DA, our alternate algorithms all increase the number of applicants admitted to college. Our alternate algorithms benefit low socioeconomic status applicants and increase geographic mobility, especially to the periphery. All comparisons also hold with respect to SP-DA, which we show, is essentially identical to SR-DA.

5.1 Winners, Losers, and the Number of Assigned Applicants

Table 3 compares our alternate algorithms to the SR-DA benchmark. Panel A shows that our alternate algorithms increase the number of assigned applicants by 1.7–2.0 percent (1,445–1,717 applicants, depending on the algorithm). The increase is concentrated in self-funded, full-time seats.

In the absence of the consensus property, it is interesting to understand how the alternate stable algorithms affect different groups of students. When comparing the result of an alternate algorithm to the benchmark allocation, we say that an applicant is a *winner* if she is assigned to a contract she ranked higher than her assignment in the benchmark allocation. The set of winners contains applicants that are assigned by our alternate algorithm but not assigned by SR-DA as well as applicants that are assigned in both cases, but our algorithm assigns them to a contract they ranked higher. Applicants in this last group can be assigned to the same program under different financial terms or to a different program. An applicant is a *loser* if she is assigned to a contract she ranked lower than her assigned contract in the benchmark assignment or if she becomes unassigned.

Panel B provides statistics on the set of winners from each of our alternate algorithms. We find that Algorithm 3 generates 5,283 winners relative to SR-DA. Approximately 44 percent of the winners were not assigned under the benchmark (i.e., are newly assigned by Algorithm 3) and a similar number are assigned to a different program from that of their benchmark assignment. The rest of the applicants are assigned to the same program under preferable financial terms (i.e., with state funding).

Panel C provides statistics on the set of losers from each of our alternate algorithms. We find that Algorithm 3 generates 4,734 losers relative to SR-DA. The majority of losers (more than 70 percent) are assigned to the same program under less favorable financial terms. The rest of the losers are assigned to another program under a contract they ranked lower or become unassigned.

Our theoretical analysis shows that the impact of our algorithms on redistribution is complex (Proposition 8). The direct effect of applying local market power is that some students lose their state funding and other students win by gaining admission to the program with state funding (instead of being unassigned or being assigned to a less desirable alternative). But, if the program already filled its capacity for self-funded students, this means that some students will lose their assignment to the unfunded contract. Additionally, displaced students (either winners or losers) can lead to chains of reassignments. This complexity is reflected in Panels B and C. Our empirical evaluation shows that, in spite of the theoretical possibility, a relatively small number of losers become unassigned or assigned to a new program.

Column (2) of Table 3 presents the results of changing SR-DA to SP-DA. As predicted by Proposition 2, SP-DA and SR-DA assign the same number of applicants. Furthermore, changing the assignment mechanism from SR-DA to SP-DA only changes the allocation of 8 applicants (approximately 0.01%). Consistent with Proposition 3, all 8 applicants prefer their SP-DA assignment.

According to Proposition 4, only the 8 applicants who strictly prefer the outcome of SP-DA to SR-DA have an incentive to misrepresent their preferences under SR-DA. This finding supports our reliance on reported preferences in our interpretation of winners and losers.

Table 3: Winners, losers, and the number of assigned applicants

| | Benchmark | | Alternate algorithms | | |
|--|--------------|--------------|----------------------|----------|----------|
| | SR-DA (1) | SP-DA (2) | 1 (3) | 2 (4) | 3 (5) |
| <i>A. Number of assigned applicants</i> | | | | | |
| Assigned to a contract | 84,130 | 84,130 | 85,575 | 85,688 | 85,847 |
| Assigned to a state-funded contract | 48,725 | 48,725 | 48,710 | 48,709 | 48,706 |
| Assigned to a self-funded contract | 35,405 | 35,405 | 36,865 | 36,979 | 37,141 |
| Assigned to a self-funded, full-time contract | 13,321 | 13,321 | 14,410 | 14,418 | 14,565 |
| <i>B. Winners</i> | | | | | |
| | | 8 | 5,056 | 4,798 | 5,283 |
| – Newly assigned | | 0 | 1,969 | 2,121 | 2,346 |
| – New program | | 8 | 2,168 | 2,282 | 2,498 |
| – Same program, preferred financial terms | | 0 | 919 | 395 | 439 |
| <i>C. Losers</i> | | | | | |
| | | 0 | 4,570 | 4,260 | 4,734 |
| – Newly unassigned | | 0 | 524 | 563 | 629 |
| – New program | | 0 | 645 | 674 | 762 |
| – Same program, less preferred financial terms | | 0 | 3,401 | 3,023 | 3,343 |

Notes: The table presents the number of assigned applicants under various stable assignments (Panel A). Panels B and C describe the number of winners and losers from changing the benchmark (SR-DA) to SP-DA and to our alternate algorithms. An applicant is a winner if she is assigned to a contract she ranked higher than her contract in the benchmark assignment. An applicant is a loser if she is assigned to a contract she ranked lower than her assigned contract in the benchmark assignment or if she becomes unassigned. Column (1) shows the benchmark assignment. Column (2) presents the SP-DA assignment. Columns (3)–(5) present our alternate algorithms’ assignments.

5.2 Characteristics of Winners and Losers

Table 4 compares the characteristics of winners and losers for Algorithm 3.²¹ On average, winners’ SES is lower than losers’. The share of disadvantaged applicants among winners is 6.1 percent relative to 2.6 percent among losers. Additionally, the average per-capita income in winners’ settlements of residence is lower, the average NABC-based SES index in their high schools is higher, and they are less likely to live in the capital or a county capital.

Table 4 also reveals that, on average, winners have lower academic achievement than losers. The average 11th-grade GPA among winners is 3.5, relative to 3.7 among losers. The difference corresponds to 0.25 $(=(3.689-3.483)/0.84)$ standard deviations of the distribution of college applicants’ 11th-grade GPA. In summary, the winners from changing the merit-based benchmark assignment mechanism to our alternate algorithm are lower-achieving low-SES applicants.

5.3 Geographic Mobility

In this section, we examine the effect of our alternate algorithms on geographic mobility. Geographic mobility is affected through winners who get newly assigned or get assigned to a new program at another location, and through the smaller number of losers who get

²¹Appendix Tables B6 and B7 show these comparisons for Algorithms 1 and 2, , respectively.

Table 4: Characteristics of winners and losers: Algorithm 3

| | Winners (1) | Losers (2) | p-values: (1)=(2) (3) |
|--|----------------|---------------|--------------------------|
| Disadvantaged | 0.061 | 0.026 | 0.000 |
| Per-capita annual gross income (1000 USD, 2007 prices) | 9.718 | 10.413 | 0.000 |
| NABC-based SES index | 0.177 | 0.305 | 0.000 |
| Capital | 0.183 | 0.282 | 0.000 |
| County capital | 0.191 | 0.214 | 0.005 |
| Town | 0.331 | 0.297 | 0.000 |
| Village | 0.295 | 0.207 | 0.000 |
| 11th-grade GPA (1-5) | 3.483 | 3.689 | 0.000 |
| Female | 0.570 | 0.556 | 0.160 |
| Number of applicants | 5,283 | 4,734 | 10,017 |

Notes: The table reports the mean values of characteristics of winners and losers from changing the benchmark (SR-DA) to Algorithm 3. An applicant is a winner if she is assigned to a contract she ranked higher than her contract in the benchmark assignment. An applicant is a loser if she is assigned to a contract she ranked lower than her assigned contract in the benchmark assignment or if she becomes unassigned. Column (3) presents p-values for the equality of mean characteristics of winners and losers.

unassigned or get assigned to a new program at another location. We find that our alternate algorithms increase geographic mobility, especially to the periphery.

Table 5 compares Algorithm 3 to the benchmark across three dimensions of geographic mobility.²² First, columns (1) and (2) focus on *movers*: applicants whose assigned study program is not located in the county where they reside. Second, columns (3) and (4) focus on applicants who get assigned to a study program in the periphery. Third, columns (5) and (6) focus on the subset of movers who move to the capital, that is, applicants who get assigned to a study program in the capital but do not reside there. Since low-SES applicants are more likely to reside outside the capital, and prestigious study programs are located in the capital, this latter dimension of geographic mobility may be interpreted as a measure of social mobility.

Columns (1) and (2) show that Algorithm 3 increases the number of movers by 1,194 (from 51,221 to 52,415, see Panel A). This increase is largely explained by the increase in the number of newly assigned applicants to a location outside their county of residence (1,463 newly assigned movers, see column (2) of Panel B). The difference is the result of 341 movers who become unassigned, an increase of 42 in the number of movers among winners who are assigned to a new program (=1,624–1,582), and an increase of 30 in the number of movers among losers who are assigned to a new program (=449–419).

Columns (3) and (4) show that the number of applicants assigned to the periphery increases by 1,228 (from 47,845 to 49,073, see Panel A). The increase in the number of applicants assigned to the periphery accounts for 72 percent of the increase in the number of assigned applicants. This finding is a consequence of the fact that most seats in the capital are filled in the benchmark assignment. Among winners, the number of applicants assigned to the periphery increases by 1,410 (=1,620+1,316–1,526), and, among losers, the number of applicants assigned to the periphery decreases by 182 (=377–265–294). Thus, unlike rural

²²Appendix Tables B8 and B9 show these comparisons for Algorithms 1 and 2, respectively.

hospitals, peripheral colleges can improve the utilization of available seats.

Columns (5) and (6) show that the number of applicants who move to the capital increases by 407 (from 22,444 to 22,851, see Panel A). Thus, applicants from the periphery account for 89 percent of the increase in the number of applicants assigned to the capital. Among winners, the number of applicants who move to the capital increases by 639 (=472+769-602), and, among losers, the number of applicants who move to the capital decreases by 232 (=183-232-183).

Table 5: Geographic mobility: Algorithm 3

| | Mover | | Assigned to periphery | | Moved to capital | |
|---------------------------|--------------|----------------|-----------------------|----------------|------------------|----------------|
| | SR-DA (1) | Algo. 3 (2) | SR-DA (3) | Algo. 3 (4) | SR-DA (5) | Algo. 3 (6) |
| <i>A. Total</i> | | | | | | |
| All assigned applicants | 51,221 | 52,415 | 47,845 | 49,073 | 22,444 | 22,851 |
| <i>B. Winners</i> | | | | | | |
| Newly assigned (N= 2,346) | – | 1,463 | – | 1,620 | – | 472 |
| New program (N= 2,498) | 1,582 | 1,624 | 1,526 | 1,316 | 602 | 769 |
| <i>C. Losers</i> | | | | | | |
| Newly unassigned (N= 629) | 341 | – | 265 | – | 183 | – |
| New program (N= 762) | 419 | 449 | 294 | 377 | 232 | 183 |

Notes: The table presents the number of movers (i.e., applicants whose assigned study program is not located in the county where they reside), of applicants assigned to the periphery (i.e., not to the capital), and of applicants who moved to the capital (i.e., applicants who get assigned to a study program in the capital but do not reside there) in the Algorithm 3 and benchmark (SR-DA) assignments. Panel A presents the total number of applicants with these characteristics. Panels B and C focus on winners and losers who are assigned to a different study program.

6 Discussion

We have shown that when students can be matched to colleges under different contractual terms, the set of stable allocations is large, leaving room for design even when market designers are committed to stability. In the market we focused on, contractual terms correspond to financial terms, but in other markets they may correspond to specialized study tracks (e.g., a business school offering management and accounting tracks) or to access to dormitories. Beyond college admissions, financial terms may correspond to different service terms in the military (Sönmez, 2013) or different levels of compensation (Niederle, 2007). Our findings open the door to many important questions.

First, we did not address the question of incentives. As currently formulated, our alternate algorithms are clearly not strategy-proof for students. For example, academically strong applicants have an incentive to report that self-funded contracts are unacceptable. *How does one implement alternative stable allocations while providing good incentives for students to reveal their preferences?* One approach would be to use hard evidence. For example, governments may use existing administrative data or facilitate the collection of information by colleges through forms like the Free Application for Federal Student Aid (FAFSA). American

college applicants who complete this federal form that determines their eligibility for federal financial aid can choose to forward the results of their application to particular colleges.

Second, policymakers may have different objectives (e.g., maximize the number of students assigned, diversity, etc.). *How does one compute a stable allocation that supports these objectives?* We have shown that—at least for the objective of maximizing the number of assigned students—this computation may be hard. Still, in college admissions markets, a long waiting period until the release of official result is a common practice, and this may allow solving realistic-size problems in reasonable time in practice (Leyton-Brown et al., 2017). Leveraging our characterization of stability (Lemma 3), integer programming methods in the spirit of Ágoston et al. (2016) and Delorme et al. (2019) are a promising direction. We leave these challenges to future research.

References

- Abdulkadiroğlu, Atila, and Tayfun Sönmez. 2003. "School Choice: A Mechanism Design Approach." *American Economic Review*, 93(3): 729–747.
- Abizada, Azar. 2016. "Stability and Incentives for College Admissions with Budget Constraints." *Theoretical Economics*, 11(2): 735–756.
- Afacan, Mustafa Oğuz. 2020. "Graduate Admission with Financial Support." *Journal of Mathematical Economics*, 87 114–127.
- Ágoston, Kolos Csaba, Péter Biró, and Iain McBride. 2016. "Integer Programming Methods for Special College Admissions Problems." *Journal of Combinatorial Optimization*, 32(4): 1371–1399.
- Akar, Hanife. 2010. "Globalization and Its Challenges for Developing Countries: The Case of Turkish Higher Education." *Asia Pacific Education Review*, 11(3): 447–457.
- Artemov, Georgy, Yeon-Koo Che, and Yinghua He. 2017. "Strategic 'Mistakes': Implications for Market Design Research." Mimeo.
- Ashlagi, Itai, Yash Kanoria, and Jacob D. Leshno. 2017. "Unbalanced Random Matching Markets: The Stark Effect of Competition." *Journal of Political Economy*, 125(1): 69–98.
- Avery, Christopher, Andrew Fairbanks, and Richard J. Zeckhauser. 2009. *The Early Admissions Game*.: Harvard University Press.
- Aygün, Orhan, and Inácio Bó. 2016. "College Admission with Multidimensional Privileges: The Brazilian Affirmative Action Case." Mimeo.
- Azevedo, Eduardo M., and Eric Budish. 2018. "Strategy-Proofness in the Large." *The Review of Economic Studies*, 86(1): 81–116.
- Azevedo, Eduardo M., and Jacob D. Leshno. 2016. "A Supply and Demand Framework for Two-Sided Matching Markets." *Journal of Political Economy*, 124(5): 1235–1268.
- Banerjee, Abhijit, Esther Duflo, Maitreesh Ghatak, and Jeanne Lafortune. 2013. "Marry for What? Caste and Mate Selection in Modern India." *American Economic Journal: Microeconomics*, 5(2): 33–72.
- Biró, Péter, Tamás Fleiner, Robert W. Irving, and David F. Manlove. 2010. "The College Admissions Problem with Lower and Common Quotas." *Theoretical Computer Science*, 411(34): 3136–3153.
- Biró, Péter. 2008. "Student Admissions in Hungary as Gale and Shapley Envisaged." Tech-

- nical Report TR-2008-291, University of Glasgow.
- Biró, Péter, and Sofya Kiselgof.** 2015. "College Admissions with Stable Score-Limits." *Central European Journal of Operations Research*, 23(4): 727–741.
- Caniglia, Alan S., and Daniel R. Porterfield.** forthcoming. "Addressing Under-Matching in College Enrollment: Toward an Economic Rationale and a Case Study." In *Design: Economic Design Responses to Inequality* (S. D. Kominers and A. Teytelboym, editors). OUP.
- Coles, Peter, and Ran I. Shorrer.** 2014. "Optimal Truncation in Matching Markets." *Games and Economic Behavior*, 87: 591–615.
- Delorme, Maxence, Sergio García, Jacek Gondzio, Joerg Kalcsics, David Manlove, and William Pettersson.** 2019. "Mathematical Models for Stable Matching Problems with Ties and Incomplete Lists." *European Journal of Operational Research*, 277(2): 426–441.
- Demange, Gabrielle, David Gale, and Marilda Sotomayor.** 1987. "A Further Note on the Stable Matching Problem." *Discrete Applied Mathematics*, 16(3): 217–222.
- DePalma, Anthony.** 1992. "M.I.T. ruled guilty in antitrust case." Sep 3, URL: www.nytimes.com/1992/09/03/us/mit-ruled-guilty-in-antitrust-case.html.
- Dubins, Lester E., and David A. Freedman.** 1981. "Machiavelli and the Gale–Shapley Algorithm." *American Mathematical Monthly*, 88(7): 485–494.
- Dur, Umut, Scott D. Kominers, Parag A. Pathak, and Tayfun Sönmez.** 2018. "Reserve Design: Unintended Consequences and the Demise of Boston’s Walk Zones." *Journal of Political Economy*, 126(6): 2457–2479.
- Dur, Umut, Parag A. Pathak, and Tayfun Sönmez.** forthcoming. "Explicit vs. Statistical Preferential Treatment in Affirmative Action: Theory and Evidence from Chicago’s Exam Schools." *Journal of Economic Theory*.
- Ehlers, Lars.** 2004. "In Search of Advice for Participants in Matching Markets Which Use the Deferred-Acceptance Algorithm." *Games and Economic Behavior*, 48(2): 249–270.
- Ehlers, Lars, and Jordi Massó.** 2007. "Incomplete Information and Singleton Cores in Matching Markets." *Journal of Economic Theory*, 136(1): 587–600.
- Ehrenberg, Ronald G.** 2009. *Tuition Rising*: Harvard University Press.
- Fleiner, Tamás.** 2003. "A Fixed-Point Approach to Stable Matchings and Some Applications." *Mathematics of Operations Research*, 28(1): 103–126.
- Gale, David, and Lloyd S. Shapley.** 1962. "College Admissions and the Stability of Marriage." *American Mathematical Monthly*, 69(1): 9–15.
- Halldórsson, Magnús M., Kazuo Iwama, Shuichi Miyazaki, and Hiroki Yanagisawa.** 2007. "Improved Approximation Results for the Stable Marriage Problem." *ACM Transactions on Algorithms (TALG)*, 3(3): , p. 30.
- Hassidim, Avinatan, Assaf Romm, and Ran I. Shorrer.** 2017a. "Need vs. Merit: The Large Core of College Admissions Markets." Mimeo.
- Hassidim, Avinatan, Assaf Romm, and Ran I. Shorrer.** 2017b. "Redesigning the Israeli Psychology Master’s Match." *American Economic Review*, 107(5): 205–209.
- Hassidim, Avinatan, Assaf Romm, and Ran I. Shorrer.** 2019. "Contracts Are Not Salaries in the Hidden-Substitutes Domain." *Economics Letters*, 181: 40–42.
- Hatfield, John W., and Fuhito Kojima.** 2010. "Substitutes and Stability for Matching with Contracts." *Journal of Economic Theory*, 145(5): 1704–1723.
- Hatfield, John W., and Scott D. Kominers.** 2015. "Hidden Substitutes." In *Proceedings of the*

- 16th ACM Conference on Economics and Computation.*, p. 37.
- Hatfield, John W., Scott D. Kominers, and Alexander Westkamp.** forthcoming. "Stability, Strategy-Proofness, and Cumulative Offer Mechanisms." *The Review of Economic Studies*.
- Hatfield, John W., and Paul R. Milgrom.** 2005. "Matching with Contracts." *American Economic Review*, 95(4): 913–935.
- Heo, Eun Jeong.** 2017. "Financial Aid in College Admissions: Need-Based versus Merit-Based." Mimeo.
- Hitsch, Günter, Ali Hortaçsu, and Dan Ariely.** 2010. "Matching and Sorting in Online Dating." *American Economic Review*, 100(1): 130–163.
- Horn, Dániel.** 2013. "Diverging Performances: The Detrimental Effects of Early Educational Selection on Equality of Opportunity in Hungary." *Research in Social Stratification and Mobility*, 32: 25–43.
- Hoxby, Caroline M.** 2000. "Benevolent Colluders? The Effects of Antitrust Action on College Financial Aid and Tuition." National Bureau of Economic Research, No. 7754.
- Immorlica, Nicole, and Mohammad Mahdian.** 2005. "Marriage, Honesty, and Stability." In *Proceedings of the Sixteenth Annual ACM-SIAM Symposium on Discrete Algorithms*. 53–62, Society for Industrial and Applied Mathematics.
- Kelso, Alexander S., and Vincent P. Crawford.** 1982. "Job Matching, Coalition Formation, and Gross Substitutes." *Econometrica*, 50(6): 1483–1504.
- Kim, Matthew.** 2010. "Early Decision and Financial Aid Competition among Need-Blind Colleges and Universities." *Journal of Public Economics*, 94(5–6): 410–420.
- Kiselgof, Sofya.** 2011. "Matching Practices for Universities – Ukraine." URL: www.matching-in-practice.eu.
- Klaus, Bettina, and Flip Klijn.** 2006. "Median Stable Matching for College Admissions." *International Journal of Game Theory*, 34(1): 1–11.
- Knuth, Donald E.** 1976. *Mariages stables et leurs relations avec d'autres problèmes combinatoires*.: Presses de l'Université de Montréal.
- Kojima, Fuhito, and Parag A. Pathak.** 2009. "Incentives and Stability in Large Two-Sided Matching Markets." *American Economic Review*, 99(3): 608–627.
- Konishi, Hideo, and M. Utku Ünver.** 2006. "Games of Capacity Manipulation in Hospital-intern Markets." *Social Choice & Welfare*, 27(1): 3–24.
- Lee, SangMok.** 2016. "Incentive Compatibility of Large Centralized Matching Markets." *The Review of Economic Studies*, 84(1): 444–463.
- Leyton-Brown, Kevin, Paul Milgrom, and Ilya Segal.** 2017. "Economics and Computer Science of a Radio Spectrum Reallocation." *Proceedings of the National Academy of Sciences*, 114(28): 7202–7209.
- Manlove, David F., Robert W. Irving, Kazuo Iwama, Suichi Miyazaki, and Yasufumi Morita.** 2002. "Hard Variants of Stable Marriage." *Theoretical Computer Science*, 276(1–2): 261–279.
- Nei, Stephen, and Bobak Pakzad-Hurson.** 2016. "Strategic Disaggregation in Matching Markets." Mimeo.
- Niederle, Muriel.** 2007. "Competitive Wages in a Match with Ordered Contracts." *American Economic Review*, 97(5): 1957–1969.
- Pakzad-Hurson, Bobak.** 2014. "Stable and Efficient Resource Allocation with Contracts."

Mimeo.

- Pathak, Parag A., Alex Rees-Jones, and Tayfun Sönmez.** 2020a. "Immigration Lottery Design: Engineered and Coincidental Consequences of H-1B Reforms." National Bureau of Economic Research, No. 26767.
- Pathak, Parag A., Alex Rees-Jones, and Tayfun Sönmez.** 2020b. "Reversing Reserves." National Bureau of Economic Research, No. 26963.
- Peranson, Jonah.** 2019. "Design and Implementation of the Genetic Counseling Admissions Match." In *Proceedings of MATCH-UP 2019, 5th International Workshop on Matching under Preferences*. 21–21.
- Rheingans-Yoo, Ross.** 2020. "Large Random Matching Markets with Localized Preference Structures Can Exhibit Large Cores." Mimeo.
- Romm, Assaf, Alvin E. Roth, and Ran I. Shorrer.** 2020. "Stability vs. No Justified Envy." Mimeo.
- Roth, Alvin E.** 1982. "The Economics of Matching: Stability and Incentives." *Mathematics of Operations Research*, 7(4): 617–628.
- Roth, Alvin E.** 1984a. "The Evolution of the Labor Market for Medical Interns and Residents: A Case Study in Game Theory." *Journal of Political Economy*, 92(6): 991–1016.
- Roth, Alvin E.** 1984b. "Stability and Polarization of Interests in Job Matching." *Econometrica*, 52(1): 47–57.
- Roth, Alvin E.** 1985. "The College Admissions Problem is not Equivalent to the Marriage Problem." *Journal of Economic Theory*, 36(2): 277–288.
- Roth, Alvin E.** 1986. "On the Allocation of Residents to Rural Hospitals: A General Property of Two-Sided Matching Markets." *Econometrica*, 54(2): 425–427.
- Roth, Alvin E.** 2002. "The Economist as Engineer: Game Theory, Experimentation, and Computation as Tools for Design Economics." *Econometrica*, 70(4): 1341–1378.
- Roth, Alvin E., and Elliott Peranson.** 1999. "The Redesign of the Matching Market for American Physicians: Some Engineering Aspects of Economic Design." *American Economic Review*, 89(4): 748–782.
- Roth, Alvin E., and Uriel G. Rothblum.** 1999. "Truncation Strategies in Matching Markets: In Search of Advice for Participants." *Econometrica*, 67(1): 21–43.
- Roth, Alvin E., and Xiaolin Xing.** 1994. "Jumping the Gun: Imperfections and Institutions Related to the Timing of Market Transactions." *American Economic Review*, 84(4): 992–1044.
- Schwarz, Michael, and M. Bumin Yenmez.** 2011. "Median Stable Matching for Markets with Wages." *Journal of Economic Theory*, 146(2): 619–637.
- Shorrer, I. Ran, and Sándor Sóvágó.** 2018. "Obvious Mistakes in a Strategically Simple College Admissions Environment: Causes and Consequences." Mimeo.
- Sönmez, Tayfun.** 1997. "Manipulation via Capacities in Two-Sided Matching Markets." *Journal of Economic Theory*, 77(1): 197–204.
- Sönmez, Tayfun.** 1999. "Can Pre-arranged Matches Be Avoided in Two-Sided Matching Markets?" *Journal of Economic Theory*, 86(1): 148–156.
- Sönmez, Tayfun.** 2013. "Bidding for Army Career Specialties: Improving the ROTC Branching Mechanism." *Journal of Political Economy*, 121(1): 186–219.
- Storms, Evan.** 2013. "Incentives and Manipulation in Large Market Matching with Substitutes." Mimeo.

- Wang, Tong, and Congyi Zhou.** 2018. "Purchasing Seats for High School Admission in China." Mimeo.
- Westkamp, Alexander.** 2013. "An Analysis of the German University Admissions System." *Economic Theory*, 53(3): 561–589.
- Yenmez, M. Bumin.** 2018. "A College Admissions Clearinghouse." *Journal of Economic Theory*, 176: 859–885.

Appendix A Proofs

Proof of Lemma 1. SR-DA's definition specifies an identical process to the college-proposing deferred acceptance algorithm of [Gale and Shapley \(1962\)](#). For SP-DA, note that on the run of the algorithm colleges never face a choice from a set that includes two contracts with the same student. Consequently, colleges' choices on the run of the algorithm coincide with the union of the choices of the two corresponding auxiliary colleges in the related market. Hence, the corresponding matching of $Y^{\text{SP-DA}}$ in the related market is the student-optimal stable matching. \square

Proof of Lemma 2. Let Y be a feasible allocation. If Y is not stable, then it must violate at least one of Conditions 1–4 of Lemma 3. If Y violates Condition 1 (Y is not individually rational), then either some student is assigned an unacceptable contract, or some college violates its quotas, or some college is assigned an unacceptable student. In all cases the corresponding matching also violates individual rationality. If Y violates Condition 2 (Y is blocked through $\{(s, c, t)\}$), then either $|Y \cap (S \times \{c\} \times \{t\})| < q_c^t$ or there exists $(s', c, t) \in Y$ such that $s \gg_c s'$. In either case, s and (c, t) block the corresponding matching in the related market. If Y violates Condition 3 (Y is blocked through $\{(s', c, 1 - t), (s, c, t)\}$ such that $(s, c, 1 - t) \notin Y$ and $(s', c, t) \in Y$), then the student s' and the auxiliary college $(c, 1 - t)$ block the corresponding matching. Finally, if an individually rational allocation, Y , violates Condition 4 (Y is blocked through Z such that $(s, c, t) \in Z$ implies $(s', c, 1 - t) \in Y$), then the corresponding matching is blocked by s and (c, t) such that $(s, c, t) \in Z$ and for all $s' \gg_c s$ $(\{s'\} \times \{c\} \times T) \cap Z = \emptyset$. \square

Proof of Lemma 3. If Y violates one of Conditions 1–4, then it is not stable. For the other direction, assume Y is not stable. We show that if Y violates none of Conditions 1, 2, or 4, then it must violate Condition 3.

Since Y is individually rational (by Condition 1), the allocation is blocked through some $Z \neq \emptyset$. By Condition 4, Z' contains a contract (\bar{s}, c, \bar{t}) such that $((\bar{s}, c, 1 - \bar{t}) \notin Y$. Furthermore, Y is also blocked through $Z' = \text{Ch}_c(Y \cup Z) \setminus Y$, which contains (\bar{s}, c, \bar{t}) .

Let $(s, c, t) \in Z'$ be a contract with the highest ranked student according to \gg_c (formally, $\hat{s} \gg_c s \implies (\{\hat{s}\} \times \{c\} \times T) \cap Z' = \emptyset$). By Condition 2, Y is not blocked through $\{(s, c, t)\}$. Therefore, the allocation Y assigns q_c^t students to c under the financial terms t , and all of them are ranked higher than s according to \gg_c . Therefore, since $(s, c, t) \in \text{Ch}(Y \cup Z')$, there must exist a contract $(s', c, 1 - t) \in Z'$ such that $(s', c, t) \in Y$. We claim that the allocation must be blocked through $\{(s, c, t), (s', c, 1 - t)\}$ (in violation of Condition 3).

To see this, note that if $|Y \cap (S \times \{c\} \times \{1 - t\})| < q_c^{1-t}$ (i.e., c 's $(1 - t)$ -quota is not full), then c would prefer to add the contract $(s, c, 1 - t)$ (s is acceptable to c since all contracts in

Z' are), and swaps in the identities of the recipients of financial aid are less important to the college than the composition of the cohort. Otherwise, both of c 's quotas are full. Thus, there exists some s'' such that $(s'', c, 1 - t) \in Y$ and $s'' \notin [Ch_c(Y \cup Z')]_S$. This holds since s (a new student) must displace someone (both quotas are full), and all students who get assigned to c under financial terms t in Y are ranked higher than s according to \gg_c (hence, for (s, c, t) to be chosen by c , a contract with each of them must also be chosen). We denote by s^* be the lowest ranked student according to \gg_c such that $(s^*, c, 1 - t) \in Y$ and $s^* \notin [Ch_c(Y \cup Z')]_S$. Finally, note that $s' \neq s^*$, and by the properties of \succ_c we have $Ch_c(Y \cup \{(s, c, t), (s', c, 1 - t)\}) = ((Y \cap (S \times \{c\} \times T)) \cup \{(s, c, t), (s', c, 1 - t)\}) \setminus \{(s', c, t), (s^*, c, 1 - t)\}$. \square

Proof of Proposition 1. Follows from Gale and Shapley (1962) by Lemmas 1 and 2. \square

Proof of Proposition 2. Follows from the rural hospital theorem for the related market. \square

Proof of Proposition 3. Follows from the consensus property in the related market. \square

Proof of Proposition 4. A student who benefits from misrepresenting her preferences to SP-DA (SR-DA) can do the same in the related market (by Lemma 1). The proposition, therefore, follows from the results of Demange et al. (1987), Dubins and Freedman (1981), and Roth (1982) for the related market. \square

Proof of Proposition 5. By Lemma 1, both $Y^{\text{SP-DA}}$ and $Y^{\text{SR-DA}}$ correspond to stable matchings in the related two-sided matching market. Hence, for each c and t , if s is matched with (c, t) and $s' \gg_c s$ then s' must weakly prefer her assignment to (c, t) . In fact, the same applies to any feasible allocation whose corresponding matching is stable. \square

Proof of Proposition 6. Follows from Example 1 in the main text and Example 2 below. \square

Example 2. There are three students, $S = \{r, p, g\}$, and two colleges, $C = \{h, c\}$. College h has two seats, but only one of these seats is state-funded ($q_h^0 = 1, q_h^1 = 1$), and college c has a single state-funded seat ($q_c^0 = 0, q_c^1 = 1$). Both colleges rank r first, p second, and g third (i.e., $r \gg_h p \gg_h g$ and $r \gg_c p \gg_c g$). Students' preferences are $(r, h, 1) \succ_r (r, h, 0) \succ_r \emptyset$, $(g, h, 1) \succ_g (g, h, 0) \succ_g \emptyset$, and $(p, h, 1) \succ_p (p, c, 1) \succ_p \emptyset$.

Under $Y^{\text{SP-DA}} = \{(r, h, 1), (p, c, 1), (g, h, 0)\}$, r and g are assigned to h , and p is assigned to c . An alternative stable allocation, $\{(r, h, 0), (p, h, 1)\}$, has the two highest-ranked students, r and p , assigned to h , while the lowest-ranked student, g , remains unassigned.

Proof of Proposition 7. See Appendix C. \square

Proof of Proposition 8. The algorithms terminate since the finite sets A and B become smaller in each iteration. If they stop while A and B are not empty, the output is stable by construction. Otherwise, it is stable as it coincides with SP-DA.

To see that the resulting stable allocation may assign more or fewer students relative to SR-DA, note that they all select the alternative stable allocations in Examples 1 and 2. \square

Additional Material for Online Publication

Appendix B Additional Tables (For Online Publication)

Section B.1 compares the realized and the benchmark assignments. Section B.2 presents additional summary statistics. Section B.3 presents empirical findings for Algorithms 1 and 2.

B.1 Data

This Appendix presents number of applicants assigned in 2007 and in our benchmark assignment (Table B1).

Table B1: Realized and benchmark assignments

| | Realized (1) | Benchmark (2) |
|-------------------------------------|-----------------|------------------|
| Assigned to a contract | 81,563 | 84,130 |
| Assigned to a state-funded contract | 48,726 | 48,725 |
| Assigned to a self-funded contract | 32,837 | 35,405 |

Notes: The table presents the number of applicants assigned under each of the financial terms in the realized assignment in 2007 (column (1)) and in our benchmark assignment (column (2)).

B.2 Summary Statistics

This appendix presents additional summary statistics. Table B2 summarizes the means and standard deviations of the background characteristics of applicants. Table B3 shows summary statistics on the characteristics of applicants' ROLs by the type of the settlement where applicants reside, i.e., by our alternate proxy for socioeconomic status (cf. Table 2).

Table B4 presents the coefficients of a linear regression of ROL characteristics on disadvantaged status and 11th-grade GPA on the sample of applicants. We find that disadvantaged applicants are more (less) likely to rank state-funded (self-funded) contracts exclusively in their ROLs. We also find that these differences cannot be explained by the differences in applicants' academic achievement, and thus, by the differences in their admission chances.

Next, we consider three alternative proxies for socioeconomic status: per-capita annual gross income, NABC-based SES index, and the type of the settlement where applicants reside (capital, county capital, town, and village). Table B5 shows that conditional on academic achievement, applicants of higher socioeconomic status are more likely to rank at least one self-funded contract in their ROL.

Table B2: Summary statistics on applicants' characteristics

| | Mean (1) | SD (2) | N (3) |
|--|-------------|-----------|----------|
| Disadvantaged | 0.05 | 0.21 | 108,854 |
| Per-capita annual gross income (1000 USD, 2007 prices) | 9.90 | 2.30 | 106,934 |
| NABC-based SES index | 0.22 | 0.37 | 84,455 |
| Capital | 0.20 | 0.40 | 106,934 |
| County capital | 0.21 | 0.41 | 106,934 |
| Town | 0.33 | 0.47 | 106,934 |
| Village | 0.26 | 0.44 | 106,934 |
| 11th-grade GPA (1-5) | 3.63 | 0.84 | 85,811 |
| Female | 0.57 | 0.50 | 108,854 |
| Number of alternatives in ROL | 3.71 | 2.21 | 108,854 |
| Number of programs in ROL (observed) | 3.01 | 1.30 | 108,854 |

Notes: The table reports mean values and standard deviations of applicant characteristics. Disadvantaged status is an indicator for claiming priority points for disadvantaged status. 11th-grade GPA is the average grades in mathematics, history, and Hungarian grammar and literature. Applicants' settlement of residence is missing for 1,920 applicants, NABC-based SES index is missing for 24,399 applicants, and 11th-grade GPA is missing for 23,043 applicants.

Table B3: Summary statistics on applicants' ROLs: Applicants' residence

| | All applicants (%) (1) | Capital (%) (2) | County town (%) (3) | Town (%) (4) | Village (%) (5) |
|--|---------------------------|--------------------|------------------------|-----------------|--------------------|
| <i>A. Preference for funding</i> | | | | | |
| State-funded contract exclusively | 51.4 | 37.1 | 52.5 | 54.3 | 58.2 |
| Self-funded contract exclusively | 18.5 | 24.0 | 18.8 | 17.5 | 15.2 |
| <i>B. Preference for study characteristics</i> | | | | | |
| Single field of study | 54.9 | 58.9 | 55.2 | 54.1 | 52.4 |
| Single major | 29.6 | 33.5 | 30.5 | 28.7 | 26.9 |
| Same study program consecutively | 15.5 | 20.5 | 14.6 | 14.3 | 13.5 |
| Same study program consecutively on the top of the ROL | 11.6 | 15.0 | 11.3 | 10.7 | 10.2 |
| <i>C. Preference for institution characteristics</i> | | | | | |
| Single program location | 49.4 | 64.5 | 50.0 | 44.4 | 43.5 |
| Single university | 35.0 | 28.8 | 43.2 | 33.7 | 34.7 |
| Single faculty | 26.3 | 24.4 | 30.4 | 25.2 | 25.8 |
| # of applicants | 108,854 | 21,731 | 22,381 | 35,290 | 27,532 |

Notes: The table reports summary statistics on applicants' ROL. Panel A shows the share of applicants who rank state-funded (self-funded) contracts exclusively. Panel B shows the share of applicants who rank exclusively contracts in a single field of study, and in a single major. Panel B also shows the share of applicants who rank the same study program with state-funding and self-funding consecutively, and who rank the same study program with state-funding and self-funding consecutively on the top of the ROL. Panel C shows the share of applicants who rank exclusively contracts that are in the same settlement (single location), at a single university (single university), and at a single faculty of a university (single university). Column (1) presents these shares for all applicants, and columns (2)–(5) report these shares for applicants residing in the capital, county towns, towns, and villages, respectively.

Table B4: Socioeconomic status, academic achievement, and ROL characteristics

| Dependent variable | Ranked state-funded contract exclusively | | | Ranked self-funded contract exclusively | | |
|-------------------------------|--|---------------------|---------------------|---|----------------------|----------------------|
| | (1) | (2) | (3) | (4) | (5) | (6) |
| Disadvantaged | 0.294*** (0.006) | | 0.277*** (0.006) | -0.172*** (0.002) | | -0.152*** (0.002) |
| 11th-grade GPA (standardized) | | 0.079*** (0.002) | 0.079*** (0.002) | | -0.032*** (0.001) | -0.032*** (0.001) |
| Mean outcome (non-disadvd.) | 0.500 | 0.500 | 0.500 | 0.192 | 0.192 | 0.192 |
| R-squared | 0.015 | 0.035 | 0.049 | 0.009 | 0.047 | 0.054 |

Notes: The table presents the coefficient of linear regressions of ROL characteristics (such as whether an applicant ranked state-funded contracts exclusively on her ROL (columns (1)–(3)) and whether an applicant ranked self-funded contracts exclusively on her ROL (columns (4)–(6))) on socioeconomic status and academic achievement. The sample includes 108,854 applicants. The regressions include indicators for missing values of standardized 11th-grade GPA (23,043 applicants). Robust standard errors are in parentheses.

***. $p < 0.01$, **. $p < 0.05$, *. $p < 0.1$.

Table B5: Socioeconomic status, academic achievement, and ROL characteristics:
Additional specifications

| Dependent variable | Ranked state-funded contract exclusively | | | Ranked self-funded contract exclusively | | |
|-------------------------------|--|----------------------|----------------------|---|----------------------|----------------------|
| | (1) | (2) | (3) | (4) | (5) | (6) |
| Income (1000 USD) | -0.028*** (0.001) | | | 0.013*** (0.001) | | |
| NABC-based SES index | | -0.098*** (0.005) | | | -0.010*** (0.003) | |
| Capital | | | -0.208*** (0.002) | | | 0.078*** (0.002) |
| County capital | | | -0.061*** (0.003) | | | 0.035*** (0.003) |
| Town | | | -0.043*** (0.003) | | | 0.023*** (0.003) |
| 11th-grade GPA (standardized) | 0.081*** (0.002) | 0.077*** (0.002) | 0.081*** (0.002) | -0.033*** (0.001) | -0.017*** (0.001) | -0.033*** (0.001) |
| Mean outcome (non-disadvd.) | 0.500 | 0.500 | 0.500 | 0.192 | 0.192 | 0.192 |
| R-squared | 0.057 | 0.088 | 0.056 | 0.055 | 0.145 | 0.053 |

Notes: The table presents the coefficient of linear regressions of ROL characteristics (such as whether an applicant ranked state-funded contracts exclusively on her ROL (columns (1)–(3)) and whether an applicant ranked self-funded contracts exclusively on her ROL (columns (4)–(6))) on socioeconomic status and academic achievement. The sample includes 108,854 applicants. The regressions include indicators for missing values of standardized 11th-grade GPA (23,043 applicants), per-capita annual gross income (1,920 applicants), NABC-based SES index (24,399 applicants), and residence (1,920 applicants). The omitted category in columns (3) and (6) is village. Robust standard errors are in parentheses.

***: $p < 0.01$, **: $p < 0.05$, *: $p < 0.1$.

B.3 Empirical Findings

This Appendix presents empirical results for Algorithms 1 and 2. Appendix Tables B6 and B7 compare the characteristics of winners to losers under Algorithms 1 and 2, respectively. Appendix Tables B8 and B9 investigate geographic mobility under Algorithms 1 and 2, respectively.

Table B6: Characteristics of winners and losers: Algorithm 1

| | Winners (1) | Losers (2) | p-values: (1)=(2) (3) |
|--|----------------|---------------|--------------------------|
| Disadvantaged | 0.057 | 0.026 | 0.000 |
| Per-capita annual gross income (1000 USD, 2007 prices) | 9.735 | 10.385 | 0.000 |
| NABC-based SES index | 0.180 | 0.306 | 0.000 |
| Capital | 0.188 | 0.277 | 0.000 |
| County capital | 0.192 | 0.217 | 0.003 |
| Town | 0.329 | 0.298 | 0.001 |
| Village | 0.291 | 0.208 | 0.000 |
| 11th-grade GPA (1-5) | 3.473 | 3.704 | 0.000 |
| Female | 0.562 | 0.553 | 0.355 |
| Number of applicants | 5,056 | 4,570 | 9,626 |

Notes: The table reports the mean values of characteristics of winners and losers from changing the benchmark (SR-DA) to Algorithm 1. An applicant is a winner if she is assigned to a contract she ranked higher than her contract in the benchmark assignment. An applicant is a loser if she is assigned to a contract she ranked lower than her assigned contract in the benchmark assignment or if she becomes unassigned. Column (3) presents p-values for the equality of mean characteristics of winners and losers.

Table B7: Characteristics of winners and losers: Algorithm 2

| | Winners (1) | Losers (2) | p-values: (1)=(2) (3) |
|--|----------------|---------------|--------------------------|
| Disadvantaged | 0.060 | 0.025 | 0.000 |
| Per-capita annual gross income (1000 USD, 2007 prices) | 9.719 | 10.399 | 0.000 |
| NABC-based SES index | 0.174 | 0.306 | 0.000 |
| Capital | 0.184 | 0.280 | 0.000 |
| County capital | 0.190 | 0.216 | 0.003 |
| Town | 0.330 | 0.293 | 0.000 |
| Village | 0.297 | 0.212 | 0.000 |
| 11th-grade GPA (1-5) | 3.475 | 3.690 | 0.000 |
| Female | 0.571 | 0.552 | 0.076 |
| Number of applicants | 4,798 | 4,260 | 9,058 |

Notes: The table reports the mean values of characteristics of winners and losers from changing the benchmark (SR-DA) to Algorithm 2. An applicant is a winner if she is assigned to a contract she ranked higher than her contract in the benchmark assignment. An applicant is a loser if she is assigned to a contract she ranked lower than her assigned contract in the benchmark assignment or if she becomes unassigned. Column (3) presents p-values for the equality of mean characteristics of winners and losers.

Table B8: Geographic mobility: Algorithm 1

| | Mover | | Assigned to periphery | | Moved to capital | |
|---------------------------|--------------|----------------|-----------------------|----------------|------------------|----------------|
| | SR-DA (1) | Algo. 1 (2) | SR-DA (3) | Algo. 1 (4) | SR-DA (5) | Algo. 1 (6) |
| <i>A. Total</i> | | | | | | |
| All assigned applicants | 51,221 | 52,249 | 47,845 | 48,876 | 22,444 | 22,804 |
| <i>B. Winners</i> | | | | | | |
| Newly assigned (N= 1,969) | – | 1,249 | – | 1,362 | – | 402 |
| New program (N= 2,168) | 1,379 | 1,415 | 1,323 | 1,147 | 521 | 660 |
| <i>C. Losers</i> | | | | | | |
| Newly unassigned (N= 524) | 279 | – | 226 | – | 143 | – |
| New program (N= 645) | 348 | 370 | 256 | 327 | 191 | 153 |

Notes: The table presents the number of movers (i.e., applicants whose assigned study program is not located in the county where they reside), of applicants assigned to the periphery (i.e., not to the capital), and of applicants who moved to the capital (i.e., applicants who get assigned to a study program in the capital but do not reside there) in the Algorithm 1 and benchmark (SR-DA) assignments. Panel A presents the total number of applicants with these characteristics. Panels B and C focus on winners and losers who are assigned to a different study program.

Table B9: Geographic mobility: Algorithm 2

| | Mover | | Assigned to periphery | | Moved to capital | |
|---------------------------|--------------|----------------|-----------------------|----------------|------------------|----------------|
| | SR-DA (1) | Algo. 2 (2) | SR-DA (3) | Algo. 2 (4) | SR-DA (5) | Algo. 2 (6) |
| <i>A. Total</i> | | | | | | |
| All assigned applicants | 51,221 | 52,315 | 47,845 | 48,955 | 22,444 | 22,825 |
| <i>B. Winners</i> | | | | | | |
| Newly assigned (N= 2,121) | – | 1,330 | – | 1,460 | – | 435 |
| New program (N= 2,282) | 1,446 | 1,494 | 1,402 | 1,211 | 539 | 694 |
| <i>C. Losers</i> | | | | | | |
| Newly unassigned (N= 563) | 311 | – | 233 | – | 167 | – |
| New program (N= 674) | 376 | 403 | 265 | 339 | 209 | 167 |

Notes: The table presents the number of movers (i.e., applicants whose assigned study program is not located in the county where they reside), of applicants assigned to the periphery (i.e., not to the capital), and of applicants who moved to the capital (i.e., applicants who get assigned to a study program in the capital but do not reside there) in the Algorithm 2 and benchmark (SR-DA) assignments. Panel A presents the total number of applicants with these characteristics. Panels B and C focus on winners and losers who are assigned to a different study program.

Appendix C Computational Complexity (For Online Publication)

We will prove that Proposition 7 holds even if colleges are restricted to offer no more than one seat under each of the financial terms. Our proof relies on a reduction—we show that an algorithm that identifies a stable (certainly stable) allocation of maximum size quickly (in running time polynomial in the size of the input) can also be used to quickly produce a solution to another problem that is known to be NP-hard. Therefore, identifying such an algorithm will prove that $P = NP$.

The NP-hard problem that we reduce is a special version of MAX-SMTI, studied in Manlove et al. (2002). The description of the problem is as follows. An instance of *the restricted stable marriage problem with incomplete lists and ties* consists of a set of n men, M , and a set of n women $W = W^\sim \cup W^\approx$, where men in M and women in W^\approx have strict preferences over agents on the other side and remaining unmatched, and women in W^\sim are indifferent between two acceptable men and find all other men unacceptable. Given such an instance, I , a matching is *weakly stable* if no agent's assignment is unacceptable and there is no pair of agents that strictly prefer one another to their assignment. The problem of deciding whether I admits a stable matching under which all women are matched with men is NP-complete (Manlove et al., 2002).

Given an instance $I = \langle M, W^\sim, W^\approx, \{\succeq_i\}_{i \in M \cup W} \rangle$ we define *the corresponding Hungarian college admissions market* as follows. The set of students is $S^I = \{s_i\}_{i \in M \cup W^\sim}$, with elements corresponding to each man and each woman in W^\sim . The set of colleges $C^I = \{c_w\}_{w \in W^\sim \cup W^\approx}$. For each $w \in W^\approx$, c_w has a single state-funded seat ($q_{c_w}^1 = 1$ and $q_{c_w}^0 = 0$), while other colleges have one state-funded seat and one self-funded seat (i.e., for all $w \in W^\sim$, $q_{c_w}^1 = 1$ and $q_{c_w}^0 = 1$).

For each $w \in W^\approx$, c_w ranks students (and the outside option) according to the corresponding woman preferences ($s_m \gg_{c_w} s_{m'}$ iff the woman prefers m to m' , $s_m \gg_{c_w} \emptyset$ iff w prefers m to her outside option, and $\emptyset \gg_{c_w} s_{w'}$ for all $w' \in W^\sim$). For each $w \in W^\sim$, c_w has three acceptable students $s_w \gg_{c_w} s_{p_w} \gg_{c_w} s_{f_w}$ where p_w and m_w are the two men acceptable to w (ranked according to some arbitrary rule). Furthermore, for each $w \in W^\sim$, s_w 's most preferred alternative is the state-funded seat in c_w followed by a self-funded seat in this college, with all other alternatives being unacceptable ($(s_w, c_w, 1) \succ_{s_w} (s_w, c_w, 0) \succ_{s_w} \emptyset$, and $\emptyset \succ_{s_w} (s_w, c, t)$ for all $c \neq c_w$ and all $t \in \{0, 1\}$).

Finally, for each $m \in M$, s_m ranks contracts according to m 's preferences (ranking the state-funded alternative over self-funded one) except for contracts corresponding to $w \in W^\sim$ where only one of the terms is acceptable (depending on the students ranking on c_w 's ranking). Formally, s_m 's preferences satisfy the following conditions:

1. for each $w \in W^\sim$, $(s_m, c_w, 1) \succ_{s_m} \emptyset$ only if s_m is ranked second according to \gg_{c_w} , and $(s_m, c_w, 0) \succ_{s_m} \emptyset$ only if s_m is ranked third according to \gg_{c_w} .
2. for each $w \in W$ and t such that the contracts (s_m, c_w, t) is not unacceptable by the first condition $(s_m, c_w, t) \succ_{s_m} \emptyset$ iff w is acceptable to m .

3. for each $w, w' \in W$ and t, t' such that the contracts (s_m, c_w, t) and $(s_m, c_{w'}, t')$ are not unacceptable by the first condition, $(s_m, c_w, t) \succ_{s_m} (s_m, c_{w'}, t')$ iff m prefers w to w' .

Lemma 4. *An instance I of the restricted stable marriage problem with incomplete lists and ties admits a stable matching under which all women are matched with men if and only if the corresponding Hungarian college admissions market admits a stable allocation of cardinality $|S^I|$ (i.e., where all students are assigned to colleges).*

Proof. Assume I admits a stable matching μ under which all women are matched with men. Consider the allocation Y consisting of the contracts $\left\{ (s_{\mu(w)}, c_w, 1) \right\}_{w \in W^\sim}$ together with the contracts $\left\{ (s_{\mu(w)}, c_w, t_{\mu(w)}) \right\}_{w \in W^\sim}$ and $\left\{ (s_w, c_w, 1 - t_{\mu(w)}) \right\}_{w \in W^\sim}$, where $t_{\mu(w)}$ guarantees that $(s_{\mu(w)}, c_w, t_{\mu(w)}) \succ_{s_{\mu(w)}} \emptyset$ for each $w \in W^\sim$.²³ Then $|Y| = |S^I|$ and Y is a (certainly) stable allocation.

The only non-trivial case to verify is that of potential blocks involving colleges with two seats (corresponding to women in W^\sim). This case follows since for each of these colleges the student involved in $(s_{\mu(w)}, c_w, t_{\mu(w)}) \succ_{\mu(w)} \emptyset$ finds matching with the college under the other financial terms unacceptable. Hence, if the college is assigned its first- and second-ranked students, the college does not prefer any other allocation that is individually rational. Furthermore, if the college is assigned its first- and third-ranked students, then the first ranked student receives state funding (her most preferred contract), and the only individually rational allocation that the college prefers would require her to receive a less preferred alternative (the self-funded seat).

In the other direction, let Y be a (certainly) stable allocation such that $|Y| = |S^I|$. For each $w \in W^\sim$ the student s_w must be assigned to c_w (under some financial terms). This holds since w ranks s_w first, and s_w only ranks the contracts with c_w as acceptable. Furthermore, since S^I is equal to the number of available seats, all seats are assigned.

The matching ν that assigns w to m if and only if $(\{s_m\} \times \{c_w\} \times T) \cap Y \neq \emptyset$ is individually rational, and it clearly matches every women to a men.²⁴ We claim that it is also weakly stable. To see this, we note that no $w \in W^\sim$ can be involved in a blocking pair, as these women get their (tied) first choice men. But a blocking involving $w \in W^\sim$ and $m \in M$ implies that Y is blocked through $\{(s_m, c_w, 1)\}$. \square

Proposition 9. *Finding a maximum-size stable (or certainly stable) allocation in Hungarian college admissions markets is NP-hard, even when colleges offer no more than one seat under each of the financial terms (i.e., $q_c^t \leq 1$ for each c and t).*

Proof. Transforming the restricted stable marriage problem with incomplete lists and ties to the corresponding Hungarian college admissions market clearly requires only a polynomial running time. An algorithms that finds a maximum-size stable (or certainly stable) allocation in Hungarian college admissions markets should find a stable allocation that matches

²³Existence of such financial terms is guaranteed since w must be acceptable to $\mu(w)$ and vice versa, by the stability of μ .

²⁴That ν is indeed a matching follows by the feasibility of Y and the fact that in any stable allocation colleges with more than one seat must be a side to a contracts with a student s_w for some $w \in W^\sim$.

all students if and only if one exists. If the algorithm is guaranteed to output a maximum-size stable allocation in polynomial time, transforming the original marriage problem and running the algorithm will give an answer in polynomial time to whether the original marriage problem admits a stable matching under which all women are matched with men (by Lemma 4). And this problem is NP-complete (Manlove et al., 2002). \square

Remark 3. MAX-SMTI is not approximable within a factor of $21/19$, unless $P = NP$ (Halldórsson et al., 2007). Using this fact and the construction used to prove Lemma 4, one can establish that the maximum-size stable (or certainly stable) allocation in Hungarian college admissions markets is not approximable within a constant factor $A > 1$, even when colleges offer no more than one seat under each of the financial terms (i.e., $q_c^t \leq 1$ for every c and t).

Appendix D Variable Description (For Online Publication)

This appendix describes the construction of variables in Table B2.

- **Disadvantaged status:** Applicants receive priority points for having a low socioeconomic background. Our administrative data report these priority points. If an applicant received priority points for this reason in any of the alternatives in her ROL, we label this applicant as disadvantaged. Source: Administrative data on college admissions.
- **NABC-based SES index:** First, we compute the NABC-based SES index for each survey respondent between 2008 and 2012 (5 years) in grade 10. We then compute the high-school-specific means over the 2008–2012 period. Finally, we merge the high-school-specific NABC-based SES index with the applicants using their high-school identifier. The high-school identifier is missing for 15,521 applicants. Source: National Assessment of Basic Competencies (NABC).
- **Per-capita annual gross income:** Settlement-level income per population. Applicants come from 2,804 different settlements. Average exchange rate in 2007: 183HUF/USD. Source: T-STAR dataset (http://adatbank.krtk.mta.hu/adatbazisok___tstar)
- **Capital, county capital, town, village:** Dummy variables for the type of settlement where the applicants reside. Source: Administrative data on college admissions.
- **11th-grade GPA:** Average grade in mathematics, Hungarian grammar and literature, and history in grade 11. Source: Administrative data on college admissions.
- **Female:** Dummy variable for being female. Source: Administrative data on college admissions.
- **Number of alternatives in ROL:** Our administrative data include information on the number of alternatives in each applicant's ROL. Source: Administrative data on college admissions.
- **Number of programs in ROL (observed):** Our administrative data report the first 6 alternatives in an applicant's ROL as well as the applicant's realized assignment, in case it was ranked lower. We compute the number of programs in an applicant's ROL based on this information. Source: Administrative data on college admissions.

Appendix E Robustness Analysis (for Online Publication)

This Appendix shows that our empirical findings are robust to the way we handle minor inconsistencies in our data. Our data contain 3,686 applicants (3.4 percent of all applicants) whose reported assignment and reported priority scores are inconsistent with stability (their reported priority score does not exceed the priority-score cutoff of their reported assignment, or there is an alternative in their ROL from which they are rejected even though their reported priority score exceeds the priority-score cutoff of this alternative). These inconsistencies largely stem from applicants having a reported priority score of zero. Our main approach, which is presented in the text, holds the assignment of applicants with such inconsistencies fixed to their reported assignment, and makes the corresponding seats unavailable to others.

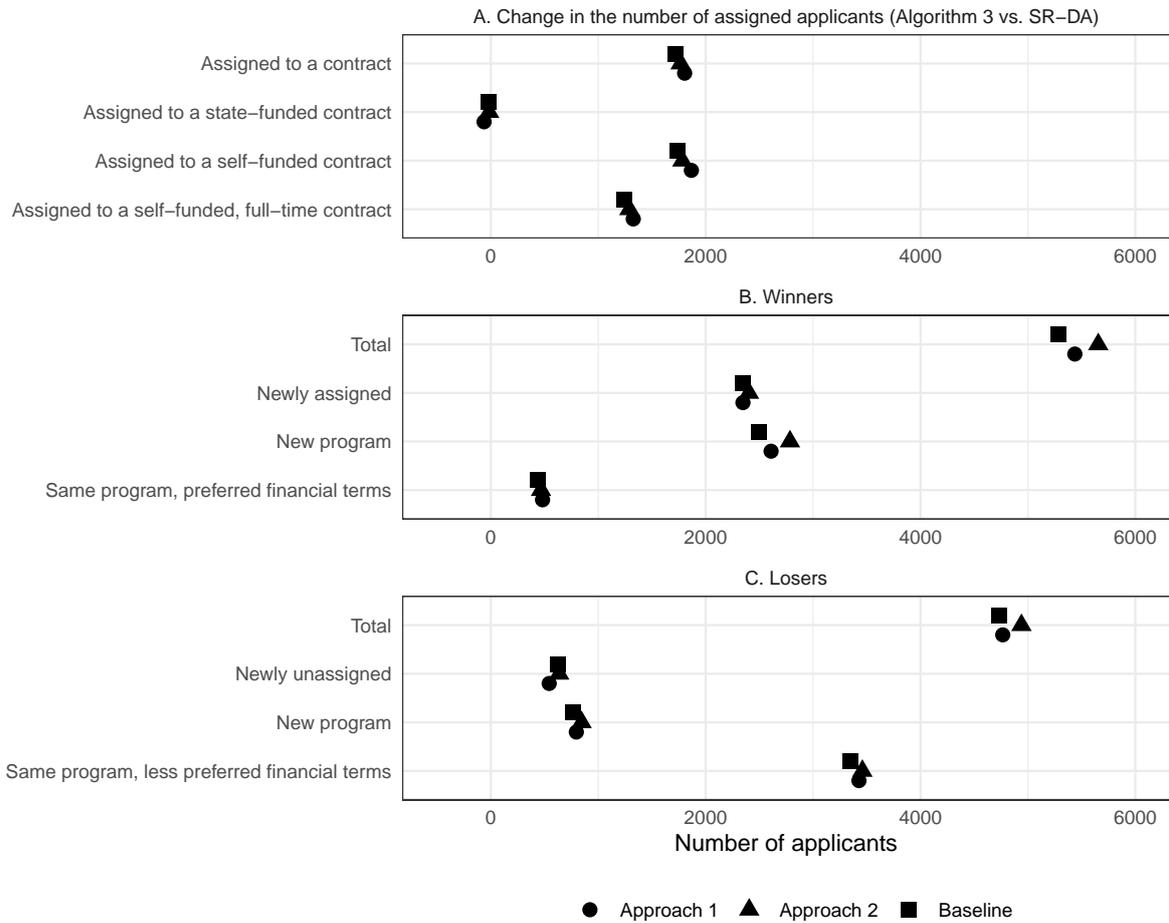
In this appendix, we consider two alternative approaches to verify that this issue does not drive our empirical findings:

- Approach 1: We hold the assignment of applicants with inconsistencies fixed to their reported assignment, but we keep the corresponding seats available to others (i.e., we do not reduce the corresponding quotas)
- Approach 2: We do not hold the assignment of applicants with inconsistencies fixed, and we keep the corresponding seats available to others.

Approach 1 changes the SR-DA (benchmark) assignment of 6,052 applicants and increases the number of assigned applicants in the benchmark by 162 relative to our main approach. Approach 2 changes the SR-DA (benchmark) assignment of 4,439 applicants and increases the number of assigned applicants in the benchmark by 1,989 relative to our main approach. In spite of these changes, Figure E1 shows that our main findings continue to hold.

Panel A of Figure E1 presents the difference between the number of assigned applicants under Algorithm 3 and under SR-DA. In the main text, this corresponds to the difference between column (5) and column (1) in Panel A of Table 3. Panel B presents the number of winners from changing the benchmark to Algorithm 3. In the main text, this corresponds to column (5) in Panel B of Table 3. Panel C presents the number of losers from changing the benchmark to Algorithm 3. In the main text, this corresponds to column (5) in Panel C of Table 3. Since the numbers in each of the panels are almost identical, we conclude that our main findings continue to hold.

Figure E1: Winners, losers, and the number of assigned applicants: Robustness



Notes: This figure studies the robustness of our empirical findings to the way we handle minor inconsistencies in our data. Panel A presents the difference between the number of assigned applicants under Algorithm 3 and under SR-DA. In the main text (baseline), this corresponds to the difference between column (5) and column (1) in Panel A of Table 3. Panel B presents the number of winners from changing the benchmark to Algorithm 3. In the main text, this corresponds to column (5) in Panel B of Table 3. Panel C presents the number of losers from changing the benchmark to Algorithm 3. In the main text, this corresponds to column (5) in Panel C of Table 3. The baseline corresponds to the analysis in the main text.